Evaluation of Temporal Spacing Errors Associated with Interval Management Algorithms

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This paper seeks to characterize the temporal spacing errors resulting from the use of Interval Management (IM) algorithms. The focus of the current paper is IM concepts and algorithms that realize a specified temporal spacing between a Target aircraft and an Ownship aircraft at the runway threshold. The paper presents an IM algorithm consisting of the following four modules: (i) Target-Landing-Time Estimation Module, (ii) Ownship-Landing-Time Estimation Module, (iii) Ownship Speed Command Computation Module, and (iv) Ownship Thrust Command Computation Module. The overall guidance module is evaluated on a simulation that models aircraft point-mass dynamics, bank-angle auto-pilot dynamics, pitch-axis auto-pilot dynamics, and engine lag dynamics. The simulation environment also consists of actual atmospheric forecasts and realistic spatio-temporally correlated wind uncertainty models. Results obtained from single case simulation as well as Monte-Carlo simulations are presented in the paper. The modeled scenario consisted of an A320 Target equipped with “Lateral Navigation”/“Vertical Navigation” (LNAV/VNAV) capabilities followed by an A320 Ownship equipped with the IM algorithm. Both aircraft fly the BIGSUR route to SFO airport using a RAP-13 1-hr wind forecast. 500 Monte-Carlo simulations were conducted with realistic wind uncertainty models. The IM algorithm for this case is seen to have a 90% probability landing time error range of 5.9 seconds, compared to the no-IM solution, which has a 90% probability landing time error range of 33.4 seconds.

I. Introduction

NASA and the FAA have been involved in extensive efforts to develop advanced concepts, technologies, and procedures for the Next Generation Air Transportation System (NextGen)¹. The objective of these research efforts has been to improve the capacity, efficiency, and safety in the next-generation National Airspace System (NAS). Improvements can come in the form of more accurate and autonomous onboard navigational capabilities based on the Global Positioning System, more accurate surveillance capabilities such as Automatic Dependent Surveillance-Broadcast (ADS-B), advanced communication capabilities such as datalink, improved vehicle designs, and improved air-traffic operations realized through advanced automation systems. A significant portion of the NextGen research is aimed at (i) developing ground-side automation systems to assist controllers in strategic planning operations, (ii) developing controller decision support tools to separate and space the traffic, and (iii) developing flight-deck-side automation to assist pilots in accomplishing airborne merging and spacing operations.

Reference 2 describes a concept for future high-density terminal air traffic operations that has been developed by the Airspace Super Density Operations (ASDO) researchers at NASA Ames Research Center. The concept described in Ref. 2 includes five core automation capabilities: 1) Extended Terminal Area Routing, 2) Precision Scheduling along Routes, 3) Merging andSpacing, 4) Tactical Separation, and 5) Off-Nominal Recovery. The first two capabilities are strategic planning tools and the remaining three are tactical decision support tools.

Successful implementation of precision scheduling requires an understanding of the following:

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• What is the range of flight times feasible for an aircraft to transit between two points along its flight path (e.g., Top of Descent to a Meterfix, Meterfix to Runway)?
• What is the accuracy with which an aircraft can realize a Scheduled Time of Arrival (STA)?

Successful realization of merging and spacing requires an understanding of the following:
• What is the accuracy with which an aircraft can maintain self-separation with respect to another aircraft?

The feasible flight time depends on the following:
• Aircraft performance characteristics
• Cruise and descent speeds selected by the Flight Management System (FMS)
• Terminal area route geometry
• Atmospheric conditions such as temperature and winds

The Time-of-Arrival (TOA) accuracy and self-separation accuracy depend on the following:
• Uncertainty associated with the atmospheric predictions.
• Advisories from ground-side controllers assisted by automation tools such as Controller Managed Spacing (CMS).
• Current-day and NextGen FMS automation capabilities listed below:
  1) LNAV & VNAV features of FMS that enable 3D-path tracking capability
  2) Required Time-of-Arrival (RTA) feature of FMS that enables an explicit TOA specification at waypoints such as the Meterfix and runway
  3) Interval Management (IM) tools that enable the capability to maintain spatial and temporal spacing with another aircraft
  4) 4-Dimensional FMS (4DFMS) capability that enables full 4D-trajectory tracking

The focus of the current research is to develop a model of IM and evaluate it in a high-fidelity simulation environment in order to establish the accuracy of its capability to maintain temporal spacing. Section II summarizes IM capabilities from published literature. Section III describes the features of the FMS-IM guidance model developed under this research. Section IV details the IM algorithm modules used in the current paper. Section V describes the simulation environment used to evaluate the IM guidance logic. Section VI presents the closed-loop simulations results. Section Error! Reference source not found. contains some concluding remarks.

II. Interval Management Capabilities

Interval Management concepts and algorithms have been an active area of research in the air-traffic management research community both in the US and Europe. The NASA Langley research of precision control for arrival operations began in the 1970s by exploring “constant distance” and “constant time” spacing techniques along a common trajectory, with onboard information and software used to enable the aircraft to independently achieve Air Traffic Control’s (ATC) operational goal. These concepts and algorithms have matured into a trajectory-based algorithm that accommodates complex route structures arriving at the airport from all directions. Aircraft speed can be based on calculations to arrive at the runway threshold at a specified time, or calculated based on achieving a specified time interval behind the preceding aircraft. In 2006, the NASA Langley research team joined the Interval Management (IM) working group, led by the Federal Aviation Administration (FAA) Surveillance Broadcast Services (SBS) Office. This group contained members from the FAA, controllers, pilots, researchers, and aircraft avionic manufacturers, and drafted a concept of operations for air traffic management procedures that would capitalize on the aircraft’s onboard ability to precisely control to an assigned spacing. The extended toolset and associated procedures are referred to as Airborne Spacing for Terminal Arrival Routes (ASTAR) and Airborne Merging and Spacing for Terminal Arrivals (AMSTAR).

IM concepts were addressed under the CoSpace project by the EUROCONTROL researchers. The objective of the CoSpace project is to determine the operational feasibility, potential benefits, and limits of the use of spacing instructions for sequencing arrival flows (Airborne Spacing Sequencing & Merging, ASPA-S&M). Airborne spacing, which involves a new allocation of tasks between controller and flight crew, is envisaged as one possible option to enhance the management of arrival flows of aircraft. It relies on the ability of the controller to task the flight crew to maintain a given spacing with respect to the preceding aircraft. The motivation is neither to transfer problems nor to give more freedom to the flight crew, but to identify a more effective task distribution beneficial to all parties without modifying responsibility for separation provision. Airborne spacing assumes air-to-air surveillance (ADS-B) along with cockpit automation (Airborne Separation Assistance System, ASAS).
Whereas the RTA feature of the FMS has significant impact on the time of arrival, the Interval Management feature of the FMS has most significant impact on the management of spacing with respect to another aircraft. Both RTA and IM use speed control to achieve their objectives; thus both these features have significant influence on the time-of-arrival. Figure 1 illustrates the functional flow and the input-output features of IM-based operations. It should be noted that the flight-deck automation requires surveillance data in addition to navigation data for continuous closed-loop control of spacing with respect to another aircraft. The nature and accuracy of the surveillance data and the range over which the data is available are important parameters of this capability.

**Figure 1. Interaction Between Ground and Flight-Deck Automation for Operations Based on Interval Management**

Table 1 provides a synopsis of the previous TOA accuracy studies conducted using different FMS IM products.

**Table 1. Synopsis of Previous IM Time-of-Arrival Accuracy/Error Studies**

<table>
<thead>
<tr>
<th>Ref. (Year)</th>
<th>Type of Study</th>
<th>Aircraft Types</th>
<th>FMS Products</th>
<th>NextGen Capability</th>
<th>Flight Phase, RTA Assignment Waypoint</th>
<th>Uncertainties</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>[25] 2010</td>
<td>Monte-Carlo Simulations</td>
<td>Diverse Aircraft Types</td>
<td>Research FMS</td>
<td>IM</td>
<td>Runway Threshold</td>
<td>Time of Arrival at Meterfix</td>
<td>95% Accuracy of 12 seconds</td>
</tr>
<tr>
<td>[26] 2006</td>
<td>Piloted Simulations</td>
<td>ASAS</td>
<td>IM</td>
<td>Descent</td>
<td>Wind</td>
<td>Time Spacing Accuracy of ± 2.5 seconds</td>
<td></td>
</tr>
<tr>
<td>[15] 2005</td>
<td>TMX Simulations</td>
<td>Diverse Aircraft Types</td>
<td>AMSTAR</td>
<td>IM</td>
<td>Descent</td>
<td>Wind</td>
<td>Time Spacing Accuracy of ± 10 seconds</td>
</tr>
</tbody>
</table>

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III. Proposed FMS-IM Model

The current section describes features of the proposed IM model. The scenario consists of two aircraft flying along the same route or different routes. The two aircraft are expected to land consecutively at the runway. The aircraft landing first is referred to as the Target and the follower aircraft is referred to as the Ownship. The IM task is posed to the Ownship. It requires the Ownship to land with a specified time spacing (e.g., 2 minutes) with respect to the Target. It is assumed that the Target is equipped with ADS-B Out and broadcasts its current position and future intent information through this channel. It is assumed that the Ownship is equipped with ADS-B (In) in order to receive updates from the Target. It is also assumed that both the Target and Ownship are equipped with LNAV/VNAV capability to enable them to fly a specified horizontal-plane path and satisfy vertical-plane constraints.

A. Inputs to the Ownship FMS

The following inputs are entered into the Ownship FMS:

- Flight plan containing the waypoint sequence
- Takeoff weight of the aircraft
- Cost Index CI
- Assigned Target flight ID and IM spacing constraints at the runway
- Atmospheric forecast in terms of wind, temperature, and pressure

Figure 2 illustrates the waypoint constraints for LNAV, VNAV, and IM modes of the FMS.

B. Functionality of the Ownship FMS

Internally, a generic FMS with LNAV, VNAV, and IM features contains the following modules.

- Navigation Module: The FMS navigation module component dynamically makes best estimates of current aircraft positions and states, as well as position accuracy, using a combination of sensor information from different sources. This information is used by other functions below.
- Performance Calculation Module: the FMS performance component provides aircraft performance information such as takeoff speeds, altitude capability, profile optimization advisories, flight time, and fuel.
- Trajectory Synthesis Module: The trajectory synthesis module computes the horizontal- and vertical-plane reference trajectory based on the flight plan, speeds, and atmospheric forecast.
• Guidance Module: The guidance module compares the navigation state of the aircraft with the reference trajectory and computes the necessary corrections in terms of bank angle command, pitch axis command, and thrust command.

• IM Module: The IM module compares the predicted time spacing with respect to a Target to compute incremental speed change commands necessary to mitigate the spacing errors.

C. Architecture of the Ownship FMS

Figure 3 shows the functional architecture of the FMS model developed under the current research. It consists of seven blocks: (i) Speed & Profile Selection, (ii) Horizontal Path Synthesis, (iii) & (iv) Horizontal and Vertical Reference Trajectory Synthesis, (v) LNAV Guidance Module, (vi) VNAV Guidance Module, and (vii) IM module.

![Functional Architecture of FMS model](image)

The guidance module in the preceding section has been decomposed into three sub-modules: (i) LNAV module, (ii) VNAV module, and the (iii) IM module.

The objective of LNAV is to ensure that the aircraft crosses each horizontal plane waypoint by actively tracking the horizontal-plane reference trajectory created by the trajectory synthesis module.

The objective of VNAV is to ensure that the aircraft tracks the vertical-plane reference trajectory.

The objective of IM module is to ensure that the aircraft lands behind the Target aircraft at the specified landing time spacing.

D. Inputs

The inputs to the guidance module can be classified into two categories: (i) inputs related to the current state of the aircraft, and (ii) inputs related to where the aircraft is expected to be. The first category of inputs is obtained from the onboard navigation/sensor systems. The second set of inputs is obtained from the FMS Reference Trajectory Synthesizer module described in the previous sections.

i. Inputs from Onboard Navigation/Sensor Systems

The onboard navigation and sensor systems measure the current state of the aircraft. They typically update the state of the aircraft at a certain update rate such as 1 Hz. These measurements provide the feedback for the aircraft to determine if corrective actions are necessary. The following is the current list of inputs to the guidance module from the onboard navigation/avionics/sensor systems:
Time stamp associated with the current navigation system update
Current latitude, longitude, and altitude, respectively, from the onboard navigation system. These could be converted into Cartesian coordinates $x_n, y_n, h_n$ with a predetermined reference as the origin. The Cartesian coordinates can further be mapped to a path length $s$ using the flight plan information.

Current true airspeed from the onboard navigation system
Current heading angle from the onboard navigation system
Current flight path angle from the onboard navigation system
Current thrust as measured by the onboard sensors
Local wind components measured by onboard sensors
Local temperature measured by onboard sensors

(ii. Inputs from FMS Reference Trajectory Synthesizer)

The FMS reference trajectory synthesizer creates a trajectory that serves as a reference for the aircraft to track. The reference trajectory is created taking into account aircraft performance characteristics, aircraft weight, engine type, atmospheric (wind & temperature) forecast, flight plan, and waypoint crossing constraints. It is assumed that the reference trajectory will consist of the following fields:

\begin{table}
\begin{tabular}{ll}
$t$ & Time \\
$s_{\text{ref}}$ & Path length \\
$x_{\text{Ref}}, y_{\text{Ref}}, h_{\text{Ref}}$ & Position coordinates \\
$V_{t,\text{ref}}$ & True airspeed \\
$V_{\text{CAS}, \text{ref}}$ & Calibrated airspeed \\
$M_{\text{ref}}$ & Mach number \\
$V_{g, \text{ref}}$ & Groundspeed \\
$\chi_{\text{ref}}$ & Heading angle \\
$\gamma_{\text{ref}}$ & Flight path angle \\
$T_{\text{ref}}$ & Thrust \\
$L_{\text{ref}}$ & Lift \\
$D_{\text{ref}}$ & Drag \\
$m_{\text{ref}}$ & Mass of the aircraft \\
$m_{f, \text{ref}}$ & Fuel mass \\
$W_{x, \text{ref}}, W_{y, \text{ref}}, W_{z, \text{ref}}$ & Forecast wind components \\
$\Theta_{\text{ref}}$ & Forecast temperature \\
$w_{p_n}$ & Next waypoint \\
\end{tabular}
\end{table}

The reference trajectory can be characterized as a matrix of the following form:

\begin{equation}
\begin{pmatrix}
0 & \ldots & \text{origin airport or cruise condition} \\
\vdots & \ddots & \vdots \\
0 & \ldots & \text{destination airport runway threshold}
\end{pmatrix}
\end{equation}

The number of columns in the above matrix will be the same as the number of fields specified previously. The number of rows depends on the total time duration associated with the trajectory and the time discretization used. The first row of the trajectory corresponds to the aircraft state at the origin airport (or at a certain cruise condition). The last row of the trajectory corresponds to the runway threshold of the destination airport. It is assumed that the reference trajectory satisfies the waypoint constraints.

(iii. Inputs from ADS-B Surveillance Sensors)

Typically aircraft transmit their horizontal plane location (latitude and longitude), altitude, speed, heading, flight path angle, and intent through the ADS-B out. IM algorithms on the Ownship use such information broadcast from a Target aircraft and estimate its landing time. This in turn requires trajectory prediction algorithms by the Ownship. The task of accurately predicting the landing time of the Target is challenging even when performed on the Target side. It becomes even more challenging on the Ownship side due to the limited information associated with the intent of the Target. Realizing this difficulty, the ASTAR framework considers the possibility of the Target broadcasting its own landing time using ADS-B every 30 seconds. In this work, it is assumed that the Target
broadcasts its own predicted landing time every 30 seconds, which is the only information that is assumed to be available from the ADS-B.

E. Outputs from the Ownship Guidance Module

The outputs of the Ownship guidance module are as follows:

a. LNAV Guidance Command: Bank angle command $\phi_{com}$

b. VNAV Guidance Command: Coefficient of lift command $C_L_{com}$

c. IM Guidance Command: Speed command $V_{com}$

F. LNAV/VNAV Guidance Modes

A separate paper (Ref. 32) addressed the development of the LNAV and VNAV guidance logic. Current paper extends it to include the IM feature.

IV. Interval Management Algorithm

The IM algorithm used in this paper is detailed in this section. It consists of four key modules: (i) Target-Landing-Time Estimation Module, (ii) Ownship-Landing-Time Estimation Module, (iii) Ownship-Speed-Command Computation Module, and (iv) Ownship-Thrust-Command Computation Module. In this work it is assumed that the Target-Landing-Time Estimation Module is implemented on the Target aircraft. It is also assumed that the Target broadcasts its expected landing time once every 30 seconds.

A. Target Landing Time Estimation

The goal of IM is to land the Ownship aircraft after a specified time spacing with the Target. For the Ownship to accomplish this goal, it is necessary for the Ownship to be aware of the Target’s landing time. A simple approach is to use the reference landing time of the Target. However, because of the uncertainty associated with the wind as well as the imperfect controls implemented on the Target, the real landing time of the target will deviate from the reference landing time.

The approach to estimate the target’s landing time $T_{L_{target est}}$ proposed in this research is based on the following equation:

$$T_{L_{target est}} = T_{L_{target ref}} + e_{TOA_{target}}$$  \hspace{1cm} (2)

where $T_{L_{target ref}}$ is the landing time of the target extracted from its reference trajectory, and $e_{TOA_{target}}$ is a correction factor based on the estimated arrival-time error. The parameter $e_{TOA_{target}}$ is defined as the time difference between the current time ($t_{current}$) and the arrival time of the Target associated with the current position ($s_{current_{target}}$) of the Target. It is computed by querying the target’s reference trajectory with the current path length:

$$e_{TOA_{target}} = t_{current} - t_{ref_{target}}(s_{current_{target}})$$ \hspace{1cm} (3)

Notice that while $T_{L_{target ref}}$ is a fixed number, $T_{L_{target est}}$ changes as the Target descends. The target landing time estimate $T_{L_{target est}}$ is assumed to be broadcast through ADS-B once every 30 seconds and received by the Ownship.

B. Ownship Landing Time Estimation

For the Ownship to land after its target in accordance with the specified interval time, it is also necessary for the Ownship to estimate its own landing time. It is proposed to estimate the Ownship’s landing time $T_{L_{ownship est}}$ using an approach similar to that of the Target:

$$T_{L_{ownship est}} = T_{L_{ownship ref}} + e_{TOA_{ownship}}$$ \hspace{1cm} (4)

where $T_{L_{ownship ref}}$ is the landing time of the Ownship from its reference trajectory, and $e_{TOA_{ownship}}$ is the time arrival error of the Ownship estimated as shown below:

$$e_{TOA_{ownship}} = t_{current} - t_{ref_{ownship}}(s_{current_{ownship}})$$ \hspace{1cm} (5)

C. Speed Command Computation

The Ownship adjusts its speed in response to the predicted landing time spacing errors. A new speed command is synthesized using the approach described in the following paragraph.
Assume the required time interval for landing the Ownship is $T_s$ behind the Target, then the required landing time for the Ownship aircraft at the runway $t_n$ is computed as:

$$t_n = T_{target\_est} + T_s$$  \text{(6)}

The predicted spacing error at the runway of the Ownship is estimated as follows:

$$t_{error} = T_{ownship\_est} - t_n$$  \text{(7)}

Using the nominal speed of the Ownship $V_r$, the speed command for the Ownship aircraft $V_{\text{com}}$ is calculated as follows

$$V_{\text{com}} = V_r + \Delta V$$ \text{(8)}

where

$$\Delta V = k_1 t_{error}$$ \text{(9)}

$k_1 = 2 \text{ft/s}^2$ is chosen as the speed computation gain for the simulation in this report. The gain is comparable and close to the gains used by the ASTAR algorithm. It should be noted that ASTAR gains vary as a function of distance from the runway.

To restrict the incremental speed command to be less than 10% (same as ASTAR) of the nominal speed, the following condition is imposed

$$|\Delta V| < g_3 V_r$$ \text{(10)}

where $g_3$ is chosen as 0.1. The IM guidance law in Equation (8) is updated every 10 seconds similar to the ASTAR specifications.

D. Thrust Command Computation

The following thrust commands are used for speed control until the approach phase:

$$\Delta T = -k_T (V - V_{\text{com}})$$ \text{(11)}

$$T_{\text{com}} = T_{idle} + \Delta T$$ \text{(12)}

The command is modified as follows in the approach phase and below:

$$T_{\text{com}} = T_{\text{approach}} + \Delta T$$ \text{(13)}

The command is modified as follows in the landing phase and below:

$$T_{\text{com}} = T_{\text{ref}} + \Delta T$$ \text{(14)}

V. Simulation Environment

The current section describes the simulation environment used for evaluating the IM + LNAV/VNAV guidance laws. The following state components are used in the simulation where the subscript $a$ denotes the variable is defined relative to wind:

$$X, Y, h, V_t, \chi_a, \gamma_a, m$$ \text{(15)}

The following atmospheric data is treated as an external input:

$$W_x, W_y, W_h, \Theta, p, \rho$$ \text{(16)}

where $\Theta, p, \rho$ denote the temperature, pressure, and density.

The following variables are treated as external controls:

$$C_L, T_{\text{com}}, D_{\text{spoiler}}, \phi$$ \text{(17)}

The bank angle $\phi$ is obtained from the LNAV guidance module. The coefficient of lift is computed by the VNAV guidance module. Thrust command is obtained from the IM module and spoiler drag is computed by the spoiler control module defined later.

The configuration of the aircraft is treated as a time-varying setting:

$$\text{configuration} \in \{\text{clean, approach, landing}\}$$ \text{(18)}

The following set of differential equations is integrated:

$$m \dot{V}_t = (T - D) - mg \sin \gamma_a$$ \text{(19)}

$$m \dot{V}_t \chi_a = L \cos \phi - mg \cos \gamma_a$$ \text{(20)}

$$m \dot{V}_t \cos \gamma_a \dot{\chi}_a = L \sin \phi$$ \text{(21)}

$$\dot{h} = V_t \sin \gamma_a$$ \text{(22)}
\[ \dot{x} = V_t \cos \gamma_a \sin \chi_a + W_x \]  \hspace{1cm} (23)
\[ \dot{y} = V_t \cos \gamma_a \cos \chi_a + W_y \]  \hspace{1cm} (24)
\[ \dot{m} = -m_{BADA}(T, V_t, h_p) \]  \hspace{1cm} (25)

where the subscript \( a \) refers to wind axes and \( W_x, W_y \) are the wind components with respect to the wind-axes. Lift \( L \) and drag \( D \) are computed as the sum of two components, one from wing and another one from spoiler.

\[ L = L_{wing} + L_{spoiler} \]  \hspace{1cm} (26)
\[ D = D_{wing} + D_{spoiler} \]  \hspace{1cm} (27)
\[ L_{wing} = \frac{1}{2} \rho V_t^2 S_{ref} C_{L_{wing}} \]  \hspace{1cm} (28)
\[ D_{wing} = \frac{1}{2} \rho V_t^2 S_{ref} C_{D_{wing}} \]  \hspace{1cm} (29)

where \( S_{ref} \) denotes the reference wing surface area.

The drag coefficient is computed as a function of the lift coefficient; and the configuration of the aircraft (clean, approach, and landing). It can be written as follows,

\[ C_{D_{wing}} = C_{D_{25BA}}(configuration) + C_{D_{BADA}}(configuration)C_{L_{wing}}^2 \]  \hspace{1cm} (30)

The following aerodynamic model for the spoiler is used. First the spoiler reference area \( S_{ref_{spoiler}} \) is selected as follows:

\[ S_{ref_{spoiler}} \]  Spoiler reference area, to be modeled as \( S_{ref_{spoiler}} = k_{spoiler} S_{ref} \cdot k_{spoiler} = 0.25 \)

The spoiler deflection angle \( \theta \) is modeled as follows:

\[ \theta \]  Spoiler deflection angle, to be modeled as \( \theta \in [0, 45^\circ] \)

The drag coefficient for the spoiler is chosen as \( C_{D_{spoiler}} = 0.1 \). The maximum drag resulting from the deployment of spoiler is computed as follows:

\[ D_{spoiler_{max}} = 1/2 \rho V_t^2 S_{ref_{spoiler}} C_{D_{spoiler}} \sin^2 \theta_{max} \]  \hspace{1cm} (31)

The spoiler deflection angle is computed as follows:

\[ \theta = \sin^{-1} \left( \frac{\min\{-D_{spoiler}, D_{spoiler_{max}}\}}{\rho V_t^2 S_{ref_{spoiler}} C_{D_{spoiler}}} \right) \]  \hspace{1cm} (32)

Notice the negative sign in front of \( D_{spoiler} \), as it is expected that \( D_{spoiler} \) is a negative quantity. The lift from spoiler deployment is computed as follows,

\[ L_{spoiler} = \frac{\rho V_t^2 S_{ref_{spoiler}} C_{L_{spoiler}} \sin 2\theta}{2} \]  \hspace{1cm} (33)

Again, notice the negative sign on the right hand side of the lift expression indicating negative lift.

Additional differential equations are used to simulate the dynamics associated with these controls. For example, the engine exhibits a lag in responding to commands, and similarly the aircraft takes some time to pitch up/down to realize the coefficient of lift. These lags are modeled using the following differential equations:

\[ \ddot{C}_L = -k_{p,CL}(C_L - C_{com}) - k_{v,CL}C_L \]  \hspace{1cm} (34)
\[ \dot{T} = -k_T(T - T_{com}) \]  \hspace{1cm} (35)
\[ \dot{\phi} = -k_{p,\phi}(\phi - \phi_{com}) - k_{v,\phi}\phi \]  \hspace{1cm} (36)

where \( C_{com}, T_{com}, \) and \( \phi_{com} \) are the guidance law commands, and the coefficients \( k_{p,CL}, k_{v,CL}, k_T, k_{p,\phi}, \) and \( k_{v,\phi} \) are designed depending on the dynamic response of the lift coefficient and engine thrust.

It should be noted that the reference trajectory synthesizer uses a lower-fidelity model. It is expected that the feedback nature of the control will accommodate the dynamics of the control actuators. As such the reference trajectory synthesizer does not account for the flight path angle \( \gamma_a \) dynamics and does not account for the lag in the thrust and the coefficient of lift. However, the guidance is evaluated in a high-fidelity simulation model to capture the effect of these modeling discrepancies. Figure 4 shows a block diagram of the closed-loop simulation environment.
Table 2 shows the Ownship and Target specifications used in this simulation. It should be noted that Target and Ownship are the same aircraft type and flying along the same route. The simulation starts when the Ownship is at 15000ft. It is assumed that the Ownship is at its reference location with respect to the Target at this initial condition.

<table>
<thead>
<tr>
<th>Aircraft type</th>
<th>A320</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airport</td>
<td>SFO</td>
</tr>
<tr>
<td>Route</td>
<td>BIGSUR</td>
</tr>
<tr>
<td>Date</td>
<td>20120717</td>
</tr>
<tr>
<td>Hour</td>
<td>10am</td>
</tr>
</tbody>
</table>

The guidance parameters used for Ownship IM simulation are summarized in Table 3.

<table>
<thead>
<tr>
<th>Target TOA Error Update Rate</th>
<th>30 seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed Command Update Rate</td>
<td>10 seconds</td>
</tr>
<tr>
<td>Limit on the Speed Control</td>
<td>10% away from the reference speed</td>
</tr>
</tbody>
</table>

Closed-loop simulation results generated with and without IM are compared to evaluate the performance of the proposed IM strategy. The main criterion is the landing time error, which is defined as the difference between the nominal landing time (target landing time plus the spacing requirement) and the Ownship actual landing time. The altitude trajectories without IM (LNAV/VNAV only) and with IM (LNAV/VNAV + IM) are compared in Figure 5 and Figure 6 respectively. The landing time spacing error without IM is 26 seconds (rounded) and with IM is 5 seconds (also rounded).
The 3D trajectory along with its horizontal path is shown in Figure 7. The North and East wind profiles used in this simulation are shown in Figure 8 and Figure 9, respectively, where the path length is defined as the horizontal distance starting from the initial position. The target landing time estimation history is plotted in Figure 10 with the estimation error shown in Figure 11. It can be seen that the closer the Target is to the runway, the better is the estimation of the landing. The predicted landing time spacing error is shown in Figure 12. The actual airspeed, commanded airspeed, and the reference airspeed are shown in Figure 13. The difference between the commanded airspeed and the reference airspeed is shown in Figure 14, which is the incremental speed changes required for
making TOA corrections. The thrust time-histories required to realize this IM performance is shown in Figure 15. It can be seen from Figure 15 that the aircraft had to deviate from the idle thrust descent strategy employed by LNAV/VNAV. The superior interval landing time performance of the IM feature comes as a result of controlling the speed using non-idle thrust settings.

Figure 7. 3D Trajectory
Figure 8. North Wind Profile

Figure 9. East Wind Profile
Figure 10. Target Landing Time Estimation

Figure 11. Target Landing Time Estimation Error
Figure 12. Ownship Landing Time Error Estimation

Figure 13. Reference, Commanded, and Actual Airspeed of Ownship
Figure 14. Incremental Speed Command

Figure 15. Thrust Profile
Figure 16 compares the cross track errors between LNAV/VNAV and LNAV/VNAV+IM. The errors in both cases are mostly less than 0.1 nmi and are compared to each other. The altitude errors shown in Figure 17 and airspeed tracking errors shown in Figure 18 are also seen to be similar for LNAV/VNAV and IM capabilities.

**Figure 16. Cross Track Error**

**Figure 17. Altitude Error**
Figure 18. Speed Error

Table 4 shows the statistics of the landing time spacing errors at the runway without IM (LNAV/VNAV only) and with IM (IM + LNAV/VNAV). The distributions of the errors are plotted in Figure 19 and Figure 20. The error distributions are compared in Figure 21. The errors with IM are clearly far superior to the case without IM.

Table 4. Statistics of Spacing Errors at Runway

<table>
<thead>
<tr>
<th>FMS Equipage</th>
<th>Mean (sec)</th>
<th>STD (sec)</th>
<th>90% Interquartile Range (sec)</th>
<th>5% Percentile (sec)</th>
<th>95% Percentile (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without IM (LNAV/VNAV only)</td>
<td>-4.0</td>
<td>10.4</td>
<td>33.4</td>
<td>-21.4</td>
<td>12.0</td>
</tr>
<tr>
<td>With IM (LNAV/VNAV)</td>
<td>7.0</td>
<td>1.8</td>
<td>5.9</td>
<td>4.0</td>
<td>9.9</td>
</tr>
</tbody>
</table>
Figure 19. Spacing Error at Runway without IM (LNAV/VNAV only)

Figure 20. Spacing Error at Runway with IM (& LNAV/VNAV)
The current paper seeks to address the following question:

‘What is the TOA uncertainty associated with an aircraft that is equipped with Interval Management algorithms?’

The paper conducted the following step-by-step approach to answer the above question:

1. Identify the functional requirements of Interval Management concepts.
2. Survey results from published literature on evaluation of Interval Management concepts.
3. Design a sample Interval Management concept and algorithm along the lines of the ASTAR algorithm. It consists of the following modules:
   a. Target landing time estimation module, which consists of logic for the Target to estimate its own landing time based on its reference trajectory and its position along the reference trajectory. It is assumed that the Target broadcasts its landing time using ADS-B every 30 seconds.
   b. Ownship landing time estimation module, which consists of logic for the Ownship to estimate its own landing time based on its reference trajectory and its TOA errors along the reference trajectory.
   c. Ownship speed computation module, which computes a speed correction command in response to the predicted landing time spacing error. The speed computation module updates its command every 10 seconds.
4. Evaluate the IM algorithm in a high-fidelity simulation environment (used for both Target and Ownship) consisting of the following:
   a. Point-mass aircraft simulation involving aircraft dynamics.
   b. BADA aircraft performance models.
   d. Wind forecast based on RAP-13 1-hr forecast models.
e. Realistic wind uncertainty models based on Optimal Synthesis Inc.’s extensive research on wind forecast error modeling.

f. Monte-Carlo simulation framework.

5. The Monte-Carlo simulation was conducted based on the following scenario:
   Scenario involves an A320 Target equipped with LNAV/VNAV and another A320 Ownship equipped with the above-described IM algorithm. Both aircraft fly along the same route, which is chosen as the BIGSUR route at SFO airport. Actual RAP-13 1-hr wind forecast from 2012, July 17, 10am was used in the simulation. 500 Monte-Carlo simulation runs were conducted with realistic wind uncertainty models. The IM algorithm for this case is seen to have 90% probability landing time error range of 5.9 seconds, compared to the no-IM solution, which has a 90% probability landing time error range of 33.4 seconds.

Work is currently underway to conduct many more such Monte-Carlo simulations using different aircraft types, flying along different routes, and using different weather forecasts to better characterize the temporal spacing accuracy of IM algorithms.

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References