Optimal Terminal Area Flow Control using Eulerian Traffic Flow Model

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Development of a terminal area flow control algorithm suitable for use as a controller decision support tool is presented. Using the available traffic data, the control algorithm determines the miles-in-trail required at the metering fixes to deliver a desired traffic flow rate on the runway based on an Eulerian model. The present research is motivated by the NASA System Oriented Runway Management program. Eulerian models have been previously advanced for use in en-route air traffic flow control. In the present work, the parameters of the Eulerian model are derived from a Bayesian estimator using real-time traffic data. The Eulerian model is then used in conjunction with optimal control theory to derive the control law. The optimality criterion is chosen as a weighted sum of the deviations between desired runway flow rates and the miles-in-trail of the aircraft arriving at the metering fixes. The tracking performance of the algorithm is illustrated for traffic arriving into the San Francisco Metroplex as well as in the Los Angeles Metroplex. Simulation results demonstrate that the proposed approach provides actionable decisions for selecting miles-in-trail control for satisfying specified airport arrival rates.

Nomenclature

SORM	System Oriented Runway Management
NextGen	Next Generation Air Transportation System
NAS	National Airspace System
TRACON	Terminal Radar Approach Control
MIT	miles-in-trail
Q-Gen	Queuing Network Model Generator
p _j (i + 1)	number of aircraft in the j^{th} control volume at time step (i+1)
$p_j(i)$	number of aircraft in the j th control volume at time step i
τ	time step of the difference equations
q_{j-1}	outflow rate of the $(j-1)^{\text{th}}$ control volume into the j^{th} control volume
q_j	outflow rate of the j th control volume
Ω_j	length of the j th control volume
v_j	average airspeed of aircraft in the j th control volume
Х	outflow rates of all the control volumes inside the terminal area
U	control
Y	outflow rates of all the runways
n	number of control volume
m	number of runways to be regulated
Е	number of entries that can apply MIT control

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$Y_{r,N}$	desired rates of the runway flow rates at a specified final sample instant
$Y_{r,k}$	flow rates of the runways at time step k
$U_{r,k}$	flow rates if no flow controls were applied
U _k	optimal flow control applied to achieve the control objectives weighting matrices specifying the relative emphasis on the terms in the performance
P, Q, R	index.
KSFO	San Francisco International Airport
KOAK	Oakland International Airport
KSJC	Mineta San Jose International Airport
KLAX	Los Angeles International Airport
KBUR	Bob Hope Airport
KSNA	John Wayne-Orange County Airport

I. Introduction

NASA's System Oriented Runway Management program (SORM) is an operational layer between surface operations and airspace management that seeks to address integrated arrival/departure, and surface operations and traffic management¹. In the Next Generation Air Transportation System (NextGen), it is expected that enhancements will be made in communication, navigation, and weather prediction, as well as traffic flow management algorithms²⁻⁴. These comprehensive set of procedures will make the National Airspace System (NAS) more stable and predictable, and will provide several opportunities for optimization. However, these benefits can only be achieved if the decisions on the air traffic control are made based on all the available information promised by NextGen in an efficient manner. SORM, as an element of enhanced traffic flow management, focuses on a systematical approach for runway management that serves to promote efficiency for the NAS, by providing decision support tools to air traffic management personnel.

The present research focus is on two major areas: firstly, statistical estimation tools have to be developed to estimate the flight times and delays between fixes in the terminal area to assist in flow control in and out of the runway; and secondly, these estimates must be used to select the runway configuration, and to synthesize runway assignment, miles-in-trail, path-stretch, hold pattern advisories to achieve traffic flow objectives at the runway. The present paper will discuss the latter. The traffic flow estimation problem and the optimal runway assignment are discussed in two companion papers^{5,6}.

This paper presents an approach for deriving an optimal control law that computes the miles-in-trail required at the metering fixes to regulate the traffic flow on the runway to meet dynamic airport acceptance rate constraints. The approach is based on an Eulerian model of the traffic flow along the arrival routes in the TRACON. The air traffic flow control problem is formulated with a performance index consisting of the integral of a quadratic form in the deviations between desired landing rates and the nominal landing rates. In order to limit the magnitude of the flow control advisories, the performance index contains a quadratic term in the difference between the nominal miles-in-trail and the modified miles-in-trail at the metering fixes.

Different levels of abstraction could be used to model the air traffic flow. The point-mass model can be used for single aircraft performance analysis but the problem dimension is proportional to the number of aircraft being considered. The lumped-parameter models such as Eulerian models can be used to describe the aggregate behavior of the traffic using lower order dynamic models. The interest in Eulerian traffic flow models arises from their success in modeling road traffic⁷⁻¹⁰, and has been used to model, analyze, and control the en-route air traffic flow¹¹⁻¹³. Simpler Eulerian models use line elements and generate air traffic models through one-dimensional control volumes, and models networks using merge and diverge nodes, whereas the more advanced versions use multiply-connected surface elements to model the traffic flow such as the center-level and sector-level air traffic models. Further research on the Eulerian traffic flow models can be found in Reference 14-18. Three major differences exist between the models developed in this paper and the early studied in these references.

1. The state variables of the Eulerian models used in this paper are the traffic flow rates instead of the aircraft counts. This approach eliminates the need to formulate the traffic flow problem as an integer problem, avoiding numerical difficulties.

- 2. The flow control problem is cast directly in terms of the miles-in-trail required at the metering fixes instead of flow rate controls which were used in the early work. This leads to more direct decision support inputs for the terminal area controllers.
- 3. The parameters of the Eulerian traffic models such as the service times are derived using a Bayesian estimator based on a queuing network abstraction⁵. The measurements are the radar tracking data. This network is constructed by using a software package developed at Optimal Synthesis Inc, termed the Queuing Network Model Generator (Q-Gen). Using Q-gen, the terminal area routes are segmented into smaller route sections called servers, with the length of each server being about 3 nmi the FAA mandated separation in the terminal area.

The remainder of the paper is organized as follows. Section II presents the general formulation of the Eulerian model. Section III adapts the basic Eulerian model for the terminal airspace, with miles-in-trail (MIT) commands at the metering fixes as the control variables. The optimal flow control law for the optimal MIT command is derived in Section IV. This control law is then applied at the San Francisco Metroplex, which includes San Francisco airport (KSFO), Oakland International Airport (KOAK), and Mineta San Jose International Airport (KSJC) in Section V. The use of the methodology is then illustrated for the Los Angeles Metroplex, consisting of Los Angeles International Airport (KLAX), Bob Hope Airport (KBUR), and John Wayne-Orange County Airport (KSNA) in Section 0. A graphical user interface (GUI) was developed to facilitate the display of the optimal miles-in-trail commands for controller's use. This GUI is described together with the overall architecture of the software package are presented in Section VIII.

II. Eulerian Model of the Open-Loop Air Traffic Flow Dynamics

In the present research, the dynamics of the air traffic flow in the terminal airspace is modeled using an Eulerian approach¹¹⁻¹², which divides the airspace into a finite number of control volumes, and invokes the conservation principle to derive a discrete-time linear dynamic model. For a one-dimensional air traffic flow, the number of aircraft in the jth control volume at time step (i+1), $p_j(i + 1)$, is equal to the number of aircraft $p_j(i)$ in the jth control volume at time step i and the difference between the number of aircraft entering the jth control volume and the number of aircraft leaving the jth control volume during a time period of τ . This relationship can be expressed succinctly by the following difference equation

$$p_j(i+1) = p_j(i) + \tau (q_{j-1}(i) - q_j(i))$$
(1)

The outflow rate is related to the number of aircraft in the control volume through the following equation $q_j = v_j p_j / \Omega_j$ (2)

Substituting Eq. (2) into (1), the discrete equation for the number of aircraft in the control volume can be computed as:

$$p_j(i+1) = \left(1 - \frac{\nu_j \tau}{\Omega_j}\right) p_j(i) + \tau q_{j-1}(i)$$
(3)

Defining the parameters

$$a_j = 1 - \frac{v_j \tau}{\Omega_j} \tag{4}$$

$$b_j = \frac{v_j}{\Omega_j} \tag{5}$$

the discrete equation for the outflow rate of each control volume is obtained as: $a_i(i+1) = b_i m_i(i+1)$

$$q_{j}(t+1) = b_{j}p_{j}(t+1) = b_{j}p_{j}(t+1) = b_{j}\left(a_{j}p_{j}(i) + \tau q_{j-1}(i)\right) = \frac{b_{j}a_{j}q_{j}(i)}{b_{j}} + \tau b_{j}q_{j-1}(i) = a_{j}q_{j}(i) + \tau b_{j}q_{j-1}(i)$$
(6)

Equation (6) states that the outflow rate of a control volume at the next time step is related the fraction a_j of the outflow rate at the previous step $q_j(i)$ and the inflow rate from the control volume $q_{j-1}(i)$, through an allied parameter τb_j .

For a specific choice of the step size,

$$\tau = \frac{v_j}{\Omega_j} \tag{7}$$

equation (6) becomes

$q_i(i+1) = q_{i-1}(i) \tag{8}$

which states the fact that for the uncontrolled air traffic flow, the outflow rate will be equal to the inflow rate for this choice of the time step.

III. Adapting the Eulerian Model to the Terminal Airspace

Based on the Eulerian model, the discrete dynamic equation of the air traffic flow can be formulated in a matrixvector form as

$$X_{k+1} = AX_k + BU_k \tag{9}$$

where $X \in \mathbb{R}^n$ are the outflow rates of all the control volumes inside the terminal area; U is the control and several types of traffic flow control variables can be defined for the terminal area traffic. The flow rate into a metering fix can be controlled by adjusting the MIT of the aircraft before they enter the terminal area. Once the aircraft is within the TRACON, the flow rates along the routes can be modulated through path-stretch commands and speed advisories, procedure turns and in extreme cases, placing the aircraft in hold patterns²². In the current paper, only MIT control is considered and the control $U \in \mathbb{R}^E$ is the inverse of the command of MIT. The control of the ith entry is defined as

$$U_i = 1/MIT_i \tag{10}$$

In Eq. (9), all the elements of matrix $A \in \mathbb{R}^{n \times n}$ are zero except

$$A(k,k) = a_k = 1 - \frac{\nu_k \tau}{\Omega_k} \tag{11}$$

$$A(k,e) = \tau b_k \tag{12}$$

Using the relationship that the open-loop MIT at the ith entry MIT_i is related to the flow rates λ_i and the velocity at the entry sever v_i through the following equation

$$MIT_i = v_i / \lambda_i \tag{13}$$

therefore,

$$B = B_1 V \tag{14}$$

In Eq. (14), all the elements of B_1 are zero except

$$B_1(k,g) = \tau b_k$$
, for g=1, 2, ...,E (15)

with the definition that

$$b_k = \frac{v_k}{\Omega_k} \tag{16}$$

and V in Eq. (14) is a diagonal matrix with the velocity at the entry severs v_i as its diagonal elements.

The output equation of the outflow rates of all the runways is defined as
$$Y_{k+1} = CX_k$$
(17)

where $Y \in \mathbb{R}^m$ $C \in \mathbb{R}^{m \times n}$.

IV. Flow-rate Control using Optimal Control Theory

The proposed optimization problem is to minimize a cost function J defined as:

$$J = \frac{1}{2} (Y_N - Y_{r,N})^T P(Y_N - Y_{r,N}) + \frac{1}{2} \sum_{k=0}^{N-1} [(Y_k - Y_{r,k})^T Q(Y_k - Y_{r,k}) + (U_k - U_{r,k})^T R(U_k - U_{r,k})^T Q(Y_k - Y_{r,k}) + (U_k - U_{r,k})^T R(U_k - U_{r,k})^T Q(Y_k - Y_{r,k}) + (U_k - U_{$$

subject to the dynamic equations defined in Eqs. (9) and (17). The variational Hamiltonian²³ is defined as:

$$H_{k} = \frac{1}{2} (CX_{N} - Y_{r,N})^{T} P(CX_{N} - Y_{r,N}) + \frac{1}{2} \sum_{k=0}^{N-1} \left[(CX_{k} - Y_{r,k})^{T} Q(CX_{k} - Y_{r,k}) + (U_{k} - U_{r,k})^{T} R(U_{k} - U_{r,k}) \right] + \lambda_{k+1}^{T} (AX_{k} + BU_{k})$$
(19)

The co-state equations are

$$\lambda_k = \frac{\partial (H_k)}{\partial X_k} = C^T Q \left(C X_k - Y_{r,k} \right) + A^T \lambda_{k+1}$$
⁽²⁰⁾

The optimality condition leads to the condition for the optimal control as

$$0 = \frac{\partial(H_k)}{\partial U_k} = R(U_k - U_{r,k}) + B^T \lambda_{k+1} \Rightarrow U_k = -R^{-1}B^T \lambda_{k+1} + U_{r,k}$$
(21)

and the transversality condition leads to the terminal boundary condition

$$\lambda_N = C^T P(CX_N - Y_N) \tag{22}$$

These equations represent a linear, two-point-boundary-value problem. It is well known²³ that a solution to this problem can be obtained using the sweep method. Since the states are linearly related to the costates at the final time, the sweep solution approach assumes that they are similarly related at every on the time instant through a timevarying, positive definite matrix S_k as:

$$\lambda_k = S_k X_k - V_k \tag{23}$$

Then at the $(k+1)^{th}$ step.

$$\lambda_{k+1} = S_{k+1} X_{k+1} - V_{k+1} \tag{24}$$

 $\lambda_{k+1} = S_{k+1} \lambda_{k+1} - v_{k+1}$ Substituting Eq. (24) into Eq. (21) and solving for the control variable lead to $U_k = -R^{-1}B^T (S_{k+1}X_{k+1} - V_{k+1}) + U_{r,k}$ (25)

Next, substituting Eq. (25) into dynamic Eq. (9) to yield

$$X_{k+1} = AX_k + B\left(-R^{-1}B^T(S_{k+1}X_{k+1} - V_{k+1}) + U_{r,k}\right)$$
(26)

which provides the expression for X_{k+1} as:

$$X_{k+1} = (I + BR^{-1}B^{T}S_{k+1})^{-1}(AX_{k} + BR^{-1}B^{T}V_{k+1} + BU_{r,k})$$
(27)

Substituting Eq. (27) into the co-state Eq. (20) yields:

$$\lambda_{k} = C^{T} Q (CX_{k} - Y_{r,k}) + A^{T} (S_{k+1} X_{k+1} - V_{k+1})$$

$$= C^{T} Q (CX_{k} - Y_{r,k}) + A^{T} (S_{k+1} X_{k+1} - V_{k+1})$$
(28)

$$=C^{T}Q(CX_{k} - Y_{r,k}) + A^{T}S_{k+1}(I + BR^{-1}B^{T}S_{k+1})^{-1}(AX_{k} + BR^{-1}B^{T}V_{k+1} + BU_{r,k}) - A^{T}V_{k+1}$$

Fa (28) with Fa (23) leads to

Equating Eq. (28) with Eq. (23) leads to

$$\lambda_{k} = S_{k}X_{k} - V_{k} = C^{T}QCX_{k} - C^{T}QY_{r,k} + A^{T}S_{k+1}(I + BR^{-1}B^{T}S_{k+1})^{-1}(AX_{k} + BR^{-1}B^{T}V_{k+1} + BUr, k - ATVk + 1$$
(29)

Since Eq.(29) must be valid at every sample, the dynamic equations for S_k can be obtained as:

$$S_{k} = C^{T}QC + A^{T}S_{k+1}(I + BR^{-1}B^{T}S_{k+1})^{-1}A$$
(30)

and the dynamic equations for V_k is

$$V_{k} = C^{T}QY_{r,k} - A^{T}S_{k+1}(I + BR^{-1}B^{T}S_{k+1})^{-1}BU_{r,k} + \{A^{T} - A^{T}S_{k-1}(I + BR^{-1}R^{T}S_{k-1})^{-1}BR^{-1}R^{T}\}V.$$
(31)

+ $\{A^T - A^T S_{k+1} (I + BR^{-1}B^T S_{k+1})^{-1}BR^{-1}B^T\}V_{k+1}$ The boundary conditions for S_k and V_k are obtained from Eqs. (23) and (24)

$$S_N = C^T P C \tag{32}$$
$$V_N = C^T P Y_N \tag{33}$$

With the boundary conditions defined in Eqs.
$$(32)$$
 and (33) , the variables S and V can be propagated from the nal sample N to the first sample.

With the foregoing, the optimal flow control law can be obtained in closed-form as:

$$U_{k} = -R^{-1}B^{T}\lambda_{k+1} + U_{r,k}$$

$$= -R^{-1}B^{T}(S_{k+1}X_{k+1} - V_{k+1}) + U_{r,k}$$

$$= -R^{-1}B^{T}S_{k+1}AX_{K} - R^{-1}B^{T}S_{k+1}BU_{k} + R^{-1}B^{T}V_{k+1} + U_{r,k}$$
(34)

Or

fi

$$U_{k} = (I + BR^{-1}B^{T}S_{k+1})^{-1}(-R^{-1}B^{T}S_{k+1}AX_{K} + R^{-1}B^{T}V_{k+1} + U_{r,k})$$
(35)

V. Case Study for San Francisco Area Metroplex

In this section, the proposed flow control methodology is applied at the San Francisco Metroplex, which includes San Francisco airport (KSFO), Oakland International Airport (KOAK), and Mineta San Jose International Airport (KSJC). Historical radar tracking data on October 2, 2010 is used to assemble the network using Q-Gen software and the service time estimates derived from a Bayesian estimator based on the radar track data is used to assemble the Eulerian model. As shown in Figure 2, the metering fixes for KSFO include POINT REYES (PYE), ANJEE, MODESTO (MOD), and way point 14; the metering fixes for KOAK include KARNN, STIKM, and way point 22; and the metering fixes for KSJC include waypoint 22 and 24. On this day, KSFO used Runway 28L and 28R for arrivals, KOAK used Runway 29 for arrivals, and KSJC used Runway 30R for arrivals.

Ten arrival routes were included as shown in Figure 2, consisting of:

- 1) Route0 PYE to 28R (KSFO)
- 2) Route 1 PYE to 28L(KSFO)
- 3) Route 2 ANJEE to 28L(KSFO)
- 4) Route 3 MOD to 28R(KSFO)
- 5) Route 4 KARNN to 29(KOAK)
- 6) Route 5 STIKM to 29(KOAK)

- 7) Route 6WP-14 to 28L(KSFO)
- 8) Route 7 WP-21 to 29(KOAK)
- 9) Route 8 WP-22 to 30R(KSJC)
- 10) Route 9 WP-24 to 30R(KSJC)

These traffic flows land on 4 runways, leading to E=10 and m=4. The queuing network model generated by Q-gen software is shown in Figure 2, which has 171 servers, leading to n=171 states in the Eulerian model. The service time for all of these servers obtained from the estimation process is shown in Figure 3. The step size of the Eulerian model is chosen as 10 seconds, which is smaller than the minimum service time among all the servers.



Figure 1. Traffic Flow into the San Francisco Area Metroplex -West Plan



Figure 2 Queuing Network Model of the San Francisco Metroplex - West Plan



Figure 3. Service Times of the San Francisco Metroplex Model

To solve for the optimal control formulated in Eq. (35), the flow rate at the initial time needs to be known. To obtain this initial condition, it is assumed that the flow rates are zero one hour before the close-loop MIT control and the nominal (open-loop) entry flow during this one hour period is applied as the system input. The integration of Eq. (9) will generate the flow rate at the initial time to conduct MIT control. This is a reasonable assumption since the flight times from the metering fixes to the runways are less than one hour. Assuming the maximum flow rates on 28R and 28L are required to be less than 20 aircraft per hour, and the maximum flow rates for 29 and 30R are 15 aircraft per hour. The close loop control results of the flow rates on the 4 runways are shown in Figure 4 to Figure 7. Both the open loop and close loop MIT commands are shown from Figure 8 to Figure 17. It may be observed that the requirement to reduce the flow rates at 28R leads to the command to increase the MIT in route 0 and route 3. And for other routes, the open loop MIT and close loop MIT are identical.







VI. Case Study for the Los Angeles Metroplex

In this section, the proposed flow control methodology is applied at the Los Angeles Metroplex, which includes Los Angeles International Airport (KLAX), Bob Hope Burbank Airport (KBUR), and John Wayne-Orange County Airport (KSNA). Historical radar tracking data on January 27, 2010 is used to build the network using Q-Gen software and the service time estimates based on the radar track data is used to assemble the Eulerian model. On this day, KLAX used Runway 25L and 24R for arrivals, KBUR used Runway 8 for arrivals, and KSNA used Runway 30R for arrivals.

Eleven arrival routes were included as shown in Figure 18, consisting of:

- 1) Route 0,WP-29 to RW08 (KBUR)
- 2) Route 1,WP-42 to WP-35(KSNA)
- 3) Route 2, WP-42 to RW08(KBUR)
- 4) Route 3, WP-43 to RW24R(KLAX)
- 5) Route 4, WP-43 to RW25L(KLAX)
- 6) Route 5,WP-39 to WP-35(KSNA)
- 7) Route 6,WP-42 to RW24R(KLAX)
- 8) Route 7, WP-45 to RW25L(KLAX)
- 9) Route 8, WP-46 to RW25L(KLAX)
- 10) Route 9, WP-46 to RW24R(KLAX)
- 11) Route 10,WP-42R to RW25L (KLAX)



Figure 18 Arrival Routes into the Los Angeles Metroplex

Assuming that the maximum flow rates on KSNA and KBUR are required to be less than 10 aircraft per hour, and the maximum flow rates for 25L and 24R are 20 aircraft per hour. The closed-loop results of the flow rates on the 4 runways are shown in Figure 19 to Figure 22. Both the open loop and closed-loop MIT for 6 of the routes that terminate at KLAX are shown from Figure 23 to Figure 28. It can be observed that by increasing the MIT for these 6 routes reduced the runway flow rate at KLAX to the required level and most of the closed loop MIT commands return back to the open loop commands.





VII. Decision Support Software for Terminal Area and Surface Traffic Control

A software package for decision support has been developed for use by terminal area controllers. The overall software package consists of three main modules: Q-Gen, <u>QU</u>euing network parameter <u>EST</u>imator (QUEST), and Decision Support System for SORM (DS3). Q-Gen module provides a GUI that accepts inputs from the user and displays the information related to real-time traffic display. The Q-Gen module also plays the role of a communication server through which QUEST and DS3 communicate with the user and relay information required to run each module. The overall architecture of the software package is illustrated in Figure 29.



Figure 29. Software Package for Generating MIT Advisories

For Optimal Terminal Area Flow Control, the inputs are from two sources:

- 1) From user:
 - a. Airport arrival rates
 - b. Time window for flow control (initial time T0 and final time Tf)
 - c. Weights (P,Q,R) in the performance index that relatively weights the deviation of the runway flow rates from the nominal flow rates at the final time Tf, the deviation of the runway flow rates from T0 to Tf, and the miles-in-trial (MIT) deviations from the nominal MIT.
- 2) From the Q-Gen module:
 - a. Mean inter-arrival time at each meter fix during the decision window (from T0 to Tf)
 - b. From the QUEST module (relayed via the Q-Gen module):Network information (number of routes, number of sever, and routes connectivity information)
 - c. Mean service time for each sever in the terminal area during the decision window

The outputs from the module are the MIT commands at each metering fix. The miles-in-trail algorithm accepts airport arrival rates as inputs, and then determines the optimal MIT at the metering fixes to achieve these AAR. The time interval for the application of the flow control, and the airports at which the AAR constraint must be applied are also entered in the GUI. MIT required to meet these flow objectives are computed every 10 seconds by the optimal control algorithm, and displayed at every minute. The horizontal axis of the MIT plots shows the time window for the MIT flow control. Figure 30 shows the MIT advisories being displayed for the San Francisco Metroplex. The corresponding display for the Los Angeles Metroplex is given in Figure 31.



Figure 30 Optimal MIT Advisories to Meet the AAR Constraints at the San Francisco Metroplex



Figure 31 Optimal MIT Advisories to Meet the AAR Constraints at the Los Angeles Metroplex

VIII. Conclusions

An approach based on an advanced Eulerian air traffic model was advanced for computing mile-in-trail required at the metering fixes using optimal control theory. The performance of the feedback law was illustrated using historic radar data for the San Francisco Area Metroplex and Los Angeles Metroplex. The simulation results indicate that the proposed approach can create actionable MIT advisories for regulating the traffic flow on the runways to

achieve specified airport acceptance rates. The flow rates along the routes can also be modulated through pathstretch commands, procedure turns and in extreme cases, placing the aircraft in hold patterns within the TRACON. Analysis of the terminal area traffic flow control including these controls will be of future interest.

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