
Xiaoli Bai¹, Sai Vaddi²
Optimal Synthesis Inc., Los Altos, CA, 94022

Yiyuan Zhao³
Simcon Technologies., Middletown, NY, 94022

This paper develops 3D-path tracking algorithms to simulate the Lateral NAVigation (LNAV) and Vertical NAVigation (VNAV) capabilities found in current day aircraft flight management systems. The LNAV/VNAV capability is realized using two modules: (i) Reference Trajectory Synthesis Module, and (ii) LNAV/VNAV Guidance Module. A separate paper deals with the Reference Trajectory Synthesis Module. The focus of the current paper is the design and evaluation of LNAV/VNAV Guidance Module. The guidance module is formulated as a reference-trajectory tracking controller. The control laws are based on single-input single-output linear feedback control principles. The outputs from the guidance module are: (i) bank-angle command, (ii) coefficient-of-lift command, (iii) thrust command, and (iv) spoiler drag command. The guidance module is evaluated on a simulation that models aircraft point-mass dynamics, bank-angle auto-pilot dynamics, pitch-axis auto-pilot dynamics, engine lag dynamics, atmospheric forecast model, and a realistic forecast uncertainty model. Test scenarios include A320 and MD82 aircraft flying along different arrival routes into San Francisco and Los Angeles International airports. Monte-Carlo simulation framework is used to estimate the time-of-arrival uncertainty associated with a A320 LNAV/VNAV flight.

I. Introduction

ASA and the FAA have been involved in extensive efforts to develop advanced concepts, technologies, and procedures for the Next Generation Air Transportation System (NextGen)¹. The objective of these research efforts has been to improve the capacity, efficiency, and safety in the next-generation National Airspace System (NAS). Improvements can come in the form of more accurate and autonomous onboard navigational capabilities based on the Global Positioning System, more accurate surveillance capabilities such as Automatic Dependent Surveillance-Broadcast, advanced communication capabilities such as datalink, improved vehicle designs, and improved air-traffic operations realized through advanced automation systems. A significant portion of the NextGen research is aimed at (i) developing ground-side automation systems to assist controllers in strategic planning operations, (ii) developing controller decision support tools to separate and space the traffic, and (iii) developing flight-deck-side automation to assist pilots in accomplishing airborne merging and spacing operations.

Reference 2 describes a concept for future high-density terminal air traffic operations that has been developed by the Airspace Super Density Operations (ASDO) researchers at NASA Ames Research Center. The concept described in Ref. 2 includes five core automation capabilities: 1) Extended Terminal Area Routing, 2) Precision Scheduling Along Routes, 3) Merging and Spacing, 4) Tactical Separation, and 5) Off-Nominal Recovery. The first two capabilities are strategic planning tools and the remaining three are tactical decision support tools.

Successful implementation of precision scheduling requires an understanding of the following:
1. The range of feasible flight times feasible for an aircraft to transit between two points along its flight path (e.g., Top of Descent to a Meterfix & Meterfix to Runway)
2. The accuracy with which an aircraft can realize a Scheduled Time of Arrival (STA)
3. The accuracy with which an aircraft can maintain self-separation with respect to a leading aircraft

¹ Research Scientist, 95 First Street, AIAA Member.
² Senior Research Scientist, 95 First Street, AIAA Member.
³ Senior Research Scientist, 95 First Street, AIAA Associate Fellow.
The feasible flight time depends on the following:
- Aircraft performance characteristics
- Cruise and descent speeds selected by the Flight Management System (FMS)
- Terminal area route geometry
- Atmospheric conditions such as temperature and winds

The Time-of-Arrival (TOA) accuracy and self-separation accuracy depend on the following:
- Uncertainty associated with the atmospheric predictions.
- Advisories from ground-side controllers assisted by automation tools such as Controller Managed Spacing (CMS).
- Current-day and NextGen FMS automation capabilities.

Current-day and NextGen FMS capabilities that affect the TOA accuracy at a Meterfix or runway are listed below:
1) Lateral NAVigation (LNAV) & Vertical NAVigation (VNAV) features of FMS that enable 3D-path tracking capability.
2) Required Time-of-Arrival (RTA) feature of FMS that enables an explicit TOA specification at waypoints such as the Meterfix and runway.
3) Interval Management (IM) tools that enable the capability to maintain spatial and temporal spacing with another aircraft.
4) 4Dimensional FMS (4DFMS) capability that enables full 4D-trajectory tracking.

The focus of the paper is to develop a model of the LNAV/VNAV features to simulate the 3D-path tracking capability of the FMS. It is desired that the model works for a wide range of: (i) aircraft type, (ii) aircraft speeds, (iii) aircraft weights, (iv) arrival routes, and (v) atmospheric forecasts. The model is meant to be used together with atmospheric uncertainty models developed in Ref. 24 to estimate the TOA accuracy associated with LNAV/VNAV equipped aircraft. This paper, along with companion papers given in Refs. 24-26, describes the overall research on the effect of LNAV/VNAV equipage on terminal operations.

The remainder of the paper is organized as follows. Section II describes the features of LNAV and VNAV and the functional architecture of the FMS used in the current research. Section III describes the LNAV/VNAV guidance logic adopted under the current research. Section IV describes a high-fidelity simulation environment used to evaluate the LNAV/VNAV guidance logic. And, finally Section V presents the closed loop simulations that evaluate the LNAV/VNAV guidance logic using the high-fidelity simulation environment.

II. LNAV/VNAV Capability

LNAV and VNAV together enable an aircraft's capability to track a 3D path in space. LNAV deals with the horizontal plane path and VNAV deals with vertical plane path. The paths are created taking into account waypoint constraints associated with the flight plan. Figure 1 shows a schematic of the LNAV and VNAV constraints.

![Figure 1. Schematic of the LNAV and VNAV Constraints](http://arc.aiaa.org/DOI: 10.2514/6.2013-4255)
LNAV and VNAV were first implemented on the B757 and B767 in 1982. The original intent of the features was for en-route navigation. Over the years, performances of both LNAV and VNAV have been enhanced and they continue to be improved as performance-based operations mature. Core to the VNAV is the flight route construction and the subsequent construction of the 4D trajectory defined by the flight route and aircraft performance limits. In particular, VNAV is responsible for planning the vertical path (via speed and altitude) of the aircraft as a function of distance along the horizontal flight path defined by the LNAV flight plan. The vertical reference trajectory reflects all speed and altitude restrictions specified in the flight plan while obeying aircraft performance limits.

In addition, the VNAV provides vertical guidance commands to fly the aircraft while following the reference vertical path, by generating and displaying speed and pitch / altitude targets. The guidance is enabled through pitch axis and throttle control. VNAV also computes guidance commands for the autopilot or flight director and autothrottle to follow the vertical profile. Pilots can either follow displayed commands manually, or use autopilots/autothrottle.

Figure 2 illustrates the functional architecture of the FMS with LNAV and VNAV features. The approach consists of a Reference Trajectory Generation Module and a Guidance Module. The Reference Trajectory Generation Module creates feasible reference trajectories that satisfy the waypoint constraints, while taking into account the aircraft’s performance characteristics. The Guidance Module tracks the reference trajectories in the presence of atmospheric disturbances. Specifically, the objective of LNAV is to ensure that the aircraft tracks the horizontal plane reference trajectory. The objective of VNAV is to ensure that the aircraft tracks the vertical plane reference trajectory. Additionally, a separate speed control mechanism is also used to maintain the right airspeed.

The inputs to the Guidance Module can be classified into two categories: (i) inputs related to the current state of the aircraft, and (ii) inputs related to where the aircraft is expected to be. The first category of inputs is obtained from the onboard navigation/sensor systems. The second set of inputs is obtained from the FMS Reference Trajectory Synthesizer Module, which is described in detail in an accompanying paper (Ref. 25).

The onboard navigation and sensor systems measure the current state of the aircraft, and typically update the state of the aircraft at a known update rate, e.g. 1 Hz. These measurements provide the feedback for the aircraft to determine if corrective actions are necessary. The following is the current list of inputs to the Guidance Module from the onboard navigation/avionics/sensor systems:

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Time stamp associated with the current navigation system update

Current latitude, longitude, and altitude from the onboard navigation system. These
could be converted into Cartesian coordinates \(x_n, y_n, h_n\) with a pre-chosen origin

setting. The Cartesian coordinates can further be mapped to a path length \(s_n\) using the
flight plan information.

Current true airspeed from the onboard navigation system

Current heading angle from the onboard navigation system

Current flight path angle from the onboard navigation system

Current thrust as measured by the onboard sensors

Local wind components measured by onboard sensors

Local temperature measured by onboard sensors

The FMS Reference Trajectory Generation Module creates a trajectory that serves as a reference for the aircraft
to track. The reference trajectory is created taking into account aircraft performance characteristics, aircraft weight,
engine type, atmospheric (wind & temperature) forecast, flight plan, and waypoint crossing constraints. It is
assumed the reference trajectory will consist of the following fields:

\[ t_n \] Time

\[ s \] Path length

\[ x, y, h \] Position coordinates

\[ V_t \] True airspeed

\[ V_{CAS} \] Calibrated airspeed

\[ M \] Mach number

\[ V_g \] Groundspeed

\[ \chi \] Heading angle

\[ \dot{\chi} \] Heading angle rate

\[ \gamma \] Flight path angle

\[ T \] Thrust

\[ L \] Lift

\[ D \] Drag

\[ C_L \] Coefficient of lift

\[ m \] Mass of the aircraft

\[ m_f \] Fuel mass

\[ W_{x_f}, W_{y_f}, W_{z_f} \] Forecast wind components

\[ \Theta_f \] Forecast temperature

\[ p \] Forecast pressure

\[ w_{ph} \] Next waypoint

The following assumptions are made about the reference trajectory:

1) It is assumed that once this reference trajectory is generated it is not further changed.

2) It is assumed that the reference trajectory satisfies the waypoint constraints.

The outputs of the guidance module are as follows:

1) LNAV: Bank angle command \(\phi_{com}\)

2) VNAV: Coefficient of lift command \(C_{L,com}\)

3) Speed Control: Thrust command \(T_{com}\) and Spoiler Drag command \(D_{com}\)

The choice of the above LNAV, VNAV, and speed control outputs are based on the ability to simulate a wide-
variety of aircraft using only open-source aircraft information such as Base of Aircraft DAtabase (BADA). The bank
angle command and coefficient of lift command act as surrogates for the real controls generated by LNAV and
VNAV. The VNAV actually generates a pitch attitude command to realize a desired change in lift. However,
implementing such a control system would require the knowledge of the dependence between coefficient-of-lift and
angle-of-attack neither of which are available for all aircraft in public domain.
III. LNAV/VNAV Guidance Logic

This section describes the LNAV/VNAV guidance logic adopted in this paper. Since the design information of LNAV/VNAV is considered proprietary by FMS manufacturers, the logic assembled in this work is based on information obtained from open aviation forums and Refs. 4-10. As such the purpose of this research is to estimate the TOA accuracy associated with LNAV/VNAV as opposed to actually deploying this LNAV/VNAV model on a real aircraft.

The following are the different modes of the LNAV/VNAV guidance module:

1) **LNAV**
   a. **LNAV Straight Line**: In this mode the bank angle is used to control the horizontal plane path of the aircraft such that the aircraft flies to along a straight line path segment.
   b. **LNAV Turn**: In this mode bank angle is used to control the horizontal plane path of the aircraft such that the aircraft maneuvers the turn segments.

2) **VNAV**
   a. **VNAV PATH**: In this mode the pitch attitude (or equivalently the coefficient of lift) is used to control the vertical plane path of the aircraft. This mode is used when the absolute speed errors are smaller than a certain threshold.
   b. **VNAV SPD**: In this mode the pitch attitude (or equivalently the coefficient of lift) is used to control the speed of the aircraft (above 10,000 ft). This mode is used when the absolute speed errors are greater than a certain threshold and is also called “Speed on Elevator”.

3) **Speed Control - Thrust**
   a. **Required Thrust**: This mode of thrust control is used when a desired descent-rate is needed: (i) during cruise where the desired descent rate is zero, and (ii) during landing where the desired descent rate corresponds to the 3 degree glide slope.
   b. **Idle Thrust**: This thrust setting is used in descent mode starting from the top-of-descent till the aircraft reaches 8000 ft altitude.
   c. **Approach Thrust**: This thrust setting is used between 8000 ft and 3000 ft.
   d. **Speed Control Thrust**: This thrust control mode is used when the speed errors fall below a negative threshold (the speed becomes too low).

4) **Speed Control - Spoiler Drag**
   This mode is used when the speed errors fall above a positive threshold (the speed becomes too high).

A. Guidance Switching Tables

The previous section described the different modes of the overall LNAV/VNAV guidance module. The transition between these modes is governed by state-dependent logic. Table 1 shows the guidance mode switching logic for the cruise segment, Table 2 shows the guidance mode switching logic for the descent segment above the Meterfix altitude, Table 3 shows the guidance mode switching logic for the descent segment above 8 Kft and below the Meterfix altitude, Table 4 shows the guidance mode switching logic for the descent segment below 8 Kft and above 3 Kft, and Table 5 shows the guidance mode switching logic for the descent segment below 3 Kft.

**Table 1. Guidance Mode Switching Matrix for Cruise Segment**

<table>
<thead>
<tr>
<th>$(V - V_{cruise}) &lt; 0$</th>
<th>$(V - V_{cruise}) &gt; -15knots$</th>
<th>$(h &lt; h_{cruise})$</th>
<th>$(h &gt; h_{cruise})$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VNAV</strong></td>
<td><strong>VNAV PATH</strong></td>
<td><strong>VNAV PATH</strong></td>
<td><strong>VNAV PATH</strong></td>
</tr>
<tr>
<td><strong>Thrust</strong></td>
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<td><strong>CRUISE THRUST</strong></td>
<td><strong>CRUISE THRUST</strong></td>
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<tr>
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<td><strong>NONE</strong></td>
<td><strong>NONE</strong></td>
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<td><strong>VNAV SPD</strong></td>
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</tr>
<tr>
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<table>
<thead>
<tr>
<th>$V - V_{\text{cruise}} &gt; 0$</th>
<th>VNAV</th>
<th>VNAV PATH</th>
<th>VNAV PATH</th>
</tr>
</thead>
<tbody>
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<td>$(V_{\text{MO}} - V)$</td>
<td>Thrust</td>
<td>CRUISE THRUST</td>
<td>CRUISE THRUST</td>
</tr>
<tr>
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<td>NONE</td>
</tr>
<tr>
<td>Configuration</td>
<td>VNAV</td>
<td>VNAV SPD</td>
<td>VNAV SPD</td>
</tr>
<tr>
<td>$(V_{\text{MO}} - V)$</td>
<td>Thrust</td>
<td>IDLE THRUST</td>
<td>IDLE THRUST</td>
</tr>
<tr>
<td>$&lt; 30\text{knots}$</td>
<td>Spoiler</td>
<td>SPOILER DRAG</td>
<td>SPOILER DRAG</td>
</tr>
<tr>
<td>Configuration</td>
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Table 2. Guidance Mode Switching Matrix for Descent Above Meterfix Altitude

<table>
<thead>
<tr>
<th>$(V - V_{\text{ref}})$</th>
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<th>VNAV PATH</th>
<th>VNAV PATH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(V_{\text{MO}} - V)$</td>
<td>Thrust</td>
<td>CRUISE THRUST</td>
<td>CRUISE THRUST</td>
</tr>
<tr>
<td>$&gt; 30\text{knots}$</td>
<td>Spoiler</td>
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<td>NONE</td>
</tr>
<tr>
<td>Configuration</td>
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<td>VNAV SPD</td>
<td>VNAV SPD</td>
</tr>
<tr>
<td>$(V_{\text{MO}} - V)$</td>
<td>Thrust</td>
<td>CRUISE THRUST</td>
<td>CRUISE THRUST</td>
</tr>
<tr>
<td>$&lt; 30\text{knots}$</td>
<td>Spoiler</td>
<td>SPOILER DRAG</td>
<td>SPOILER DRAG</td>
</tr>
<tr>
<td>Configuration</td>
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</tbody>
</table>

Table 3. Guidance Mode Switching Matrix for Descent Above 8 Kft and Below Meterfix Altitude
Table 4. Guidance Mode Switching Matrix for Descent Above 3 Kft and Below 8 Kft

<table>
<thead>
<tr>
<th>Condition</th>
<th>(h &lt; (h_{ref}))</th>
<th>(h &gt; (h_{ref}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((V - V_{ref})) &lt; 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V - V_{ref}) &gt; (-15\text{knots})</td>
<td>VNAV</td>
<td>VNAV PATH</td>
</tr>
<tr>
<td></td>
<td>Thrust</td>
<td>APPROACH</td>
</tr>
<tr>
<td></td>
<td>Spoiler</td>
<td>NONE</td>
</tr>
<tr>
<td>((V - V_{ref})) &lt; (-15\text{knots})</td>
<td>VNAV</td>
<td>VNAV PATH</td>
</tr>
<tr>
<td></td>
<td>Thrust</td>
<td>SPEED CONTROL</td>
</tr>
<tr>
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</tr>
<tr>
<td>VNAV</td>
<td>VNAV PATH</td>
<td>VNAV PATH</td>
</tr>
<tr>
<td>((V_{MO} - V)) &gt; 30\text{knots})</td>
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<td>VNAV PATH</td>
</tr>
<tr>
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<td>APPROACH</td>
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<tr>
<td>((V_{MO} - V)) &lt; 30\text{knots})</td>
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<td>VNAV PATH</td>
</tr>
<tr>
<td></td>
<td>Thrust</td>
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</tr>
<tr>
<td></td>
<td>Spoiler</td>
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<tr>
<td>Configuration</td>
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Table 5. Guidance Mode Switching Matrix for Descent Below 3 Kft

<table>
<thead>
<tr>
<th>Condition</th>
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<th>(h &gt; (h_{ref}))</th>
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<tr>
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<tr>
<td>((V - V_{ref})) &gt; 0</td>
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<td></td>
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<tr>
<td>(V - V_{ref}) &lt; 0</td>
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<td>VNAV PATH</td>
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<td>(V - V_{ref}) &gt; 0</td>
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<tr>
<td>Configuration</td>
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</table>

B. LNAV Guidance Logic

LNAV deals with the horizontal plane guidance. Figure 3 shows a schematic of the horizontal plane guidance scenario. The flight plan waypoints are shown in blue. The reference trajectory is shown in red. The aircraft's current location is shown using a black triangle. The objective of LNAV guidance is to regulate the cross-track errors and heading error using bank angle as the control. Bank angle is used as the control for ease of modeling. The real control could be bank-angle or a angle-of-sideslip realized through rudder deflection.

![Figure 3. Schematic of the Horizontal Plane Guidance](image-url)
The discrepancy between the current horizontal plane state and the expected horizontal plane state from the reference trajectory is the basis for initiating guidance actions. Specifically, the cross-track error ($\Delta C$) and track (heading) error ($\Delta \chi$) are used as to assess the magnitude of guidance actions:

$$\Delta C = \Delta C(x_c, y_c, s_c)$$

$$\Delta \chi(s_c) = \chi_c(s_c) - \chi_{ref}(s_c)$$

where the subscripts ‘c’ and ‘ref’ refer to ‘current’ and ‘reference at current’ respectively. The current states are meant to be the same as those measured by the navigation system. It should be noted that the path-length variable $s$ is treated as the independent variable for the reference trajectory. The following guidance law is proposed for bank angle control,

$$\phi = \phi_{ref} + k_1 \Delta C + k_2 V_{g,ref} \Delta \chi$$

where $k_1$ and $k_2$ are the control gains and $V_{g,ref}$ is the ground speed.

The process of computing the cross-track error can be described as follows:

1) Inputs: The reference trajectory $(s_{ref}, x_{ref}, y_{ref})$ and the current position of the aircraft $(x_c, y_c)$
2) Step 1: Compute the perpendicular projection $(x_p, y_p)$ of the aircraft's current position on the reference trajectory.
3) Step 2: Identify the path length $s_c$ associated with $(x_p, y_p)$ by interpolation with the reference trajectory.
4) Step 3: Compute the cross-track error using the following equation:

$$\Delta C = (x_c - x_p) \cos \chi - (y_c - y_p) \sin \chi$$

where $\Delta C$ represents the cross-track error.

The heading angle error is computed as follows:

$$\Delta \chi(s_c) = \chi_c(s_c) - \chi_{ref}(s_c)$$

C. VNAV Guidance Logic

VNAV deals with vertical plane guidance which is intricately linked to speed control. Thus, VNAV has two modes: (i) VNAV PATH, and (ii) VNAV SPD. The mode VNAV PATH tracks the vertical plane path created by the reference trajectory generator. The errors are characterized in terms of altitude and altitude rate. The mode VNAV SPD is used to control large speed errors. In both modes the coefficient of lift, which is proportional to the change in pitch attitude, is used as the control. The coefficient of lift is in turn modeled as the sum of a nominal component $C_{L, nom}$ and a control component $\Delta C_L$:

$$C_L = C_{L, nom} + \Delta C_L$$

The nominal $C_L$ is computed as follows:

$$L \cos \phi = mg \cos \gamma$$

$$C_{L, nom} = \frac{2mg \cos \gamma}{\rho V^2 S_{ref} \cos \phi}$$

where $g$ is the acceleration due to gravity; $S_{ref}$ is the aerodynamic reference area; and $\rho$ is the atmospheric density.

The following control law is proposed for VNAV PATH:

$$\Delta C_L = -\frac{C_{L, nom, ref}}{g} (k_p \Delta h + k_v \Delta \dot{h})$$

where $k_p$ and $k_v$ are the control law gains.

The magnitude of the $\Delta C_L$ is constrained to be within 30% of the nominal $C_L$,

$$-0.3 C_{L, nom} \leq \Delta C_L \leq 0.3 C_{L, nom}$$

The VNAV SPD mode is invoked when the speed errors become large. In this scenario it is deemed no longer important to track the vertical plane trajectory. Instead the pitch attitude (or equivalently the coefficient of lift) is used to control speed errors ($\Delta V_t$). The following control law is proposed:

$$\Delta C_L = \frac{\cos \phi}{g \gamma_{D2, ref}} \Delta V_t$$

D. Thrust Guidance Strategies

The basis for thrust control are the errors in airspeed defined as follows:

$$\Delta V(h) = V(h) - V_{ref}(h), \text{if } V(h) < V_{ref}(h)$$

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\[ \Delta V(h) = V(h) - V_{M0} \text{ if } V(h) > V_{ref}(h) \]  

(13)

The cruise thrust computation is based on the countering the required drag to maintain \( V_t = 0 \):

\[ V_t = \frac{T - D}{m} - g \sin \gamma_a \]  

(14)

\[ T = D + mg \sin \gamma_a + k_T(V - V_{cruise}) \]  

(15)

In the above control law a feedback component is added to render the closed loop system stable. It is expected that this term should be very small.

The idle thrust is computed as a function of the airspeed, altitude, and temperature:

\[ T = T_{idle}(V_t, h, \Theta) \]  

(16)

The approach thrust is also computed as a function of the airspeed, altitude, and temperature:

\[ T = T_{approach}(V_t, h, \Theta) \]  

(17)

The required thrust is computed such that a desired altitude rate \( \dot{h} \) is realized. This is shown as follows:

\[ \dot{V}_t = \frac{T - D}{m} - g \sin \gamma_a \]  

(18)

\[ \frac{dV}{dh} \frac{dh}{dt} = \frac{T - D}{m} - g \sin \gamma_a \]  

(19)

\[ \frac{dV}{dh} \frac{dh}{dt} = \frac{T - D}{m} - g \sin \gamma_a \]  

(20)

\[ \frac{dV}{dh} \frac{dh}{dt} = \frac{T - D}{m} - g \sin \gamma_a \]  

(21)

\[ T = \frac{mg \sin \gamma_a}{V_t} \left(1 + \frac{\dot{V}_t}{g \frac{dV}{dh}}\right) + D \]  

(22)

The following control law is proposed for speed control:

\[ T = T_{idle} + \Delta T \]  

(23)

\[ \Delta T = -k_T \Delta V \]  

(24)

E. Spoiler Guidance Strategy

Spoilers are deployed when the airspeeds become very high and near the maximum speed limits. Spoiler drag is used to decelerate the aircraft in this case. The total drag \( D \) is modeled as the sum of a nominal drag resulting from the drag polar and the spoiler drag \( D_{spoiler} \):

\[ D = D_{nom} + D_{spoiler} \]  

(25)

\[ D_{nom} = C_{D0} + C_{D2} \frac{\dot{p}^2}{\dot{V}} \]  

where \( C_{D0} \) and \( C_{D2} \) are the drag-polar coefficients.

The following control law is proposed for the spoiler drag:

\[ D_{spoiler} = -k_D \Delta V \]

IV. Simulation Environment

The current section describes the simulation environment used for evaluating the LNAV/VNAV guidance laws.

A. Point-Mass Aircraft Simulation Environment

The following state components are used in the simulation:

State components \( X, Y, h, V_t, \chi_a, \gamma_a, m \)  

(26)

where \( X, Y \) are the position coordinates in a Cartesian frame of reference; \( h \) is the altitude; \( \chi_a \) is the air-relative heading angle; \( \gamma_a \) is the air-relative flight path angle; and \( m \) is the mass of the aircraft. The following atmospheric data is treated as an external input:

Atmospheric inputs \( W_x, W_y, W_h, \Theta, p, \rho \)  

(27)
where $W_x, W_y, W_h$ are the wind components; $\Theta$ is the temperature; $p$ is the pressure; and $\rho$ is the density. The atmospheric data is modeled as the sum of two components: (i) atmospheric forecast, and (ii) atmospheric uncertainty. The atmospheric forecast for the simulations is based on National Oceanic and Atmospheric Administration’s (NOAA’s) Rapid Update Cycle data (RUC). The atmospheric uncertainty model is based on Ref. 24 which models the altitude dependent variation of wind-forecast errors as well as the spatial & temporal correlation of the wind-forecast errors.

The following variables are treated as external controls:

Controls:

$$C_{r,T,D_{spoiler}, \phi}$$ \hspace{1cm} (28)

The bank angle $\phi$ is obtained from the LNAV guidance module. The coefficient of lift is computed by the VNAV guidance module. Thrust and spoiler drag are computed by the thrust control module.

The configuration of the aircraft is treated as a time-varying setting:

Time-Varying Settings $configuration \in \{\text{clean, approach, landing}\}$ \hspace{1cm} (29)

The following set of differential equations serves as the model for system dynamics, which is integrated as part of the simulation:

$$m \ddot{V}_t = (T - D) - mg \sin \gamma_a$$ \hspace{1cm} (30)

$$mV_t \dot{\gamma}_a = L \cos \phi - mg \cos \gamma_a$$ \hspace{1cm} (31)

$$mV_t \cos \gamma_a \dot{x}_a = L \sin \phi$$ \hspace{1cm} (32)

$$\dot{h} = V_t \sin \gamma_a$$ \hspace{1cm} (33)

$$\dot{x} = V_t \cos \gamma_a \sin \chi_a + W_x$$ \hspace{1cm} (34)

$$\dot{y} = V_t \cos \gamma_a \cos \chi_a + W_y$$ \hspace{1cm} (35)

$$\dot{m} = -\rho_{\text{BADA}}(T, V_t, h_p)$$ \hspace{1cm} (36)

where the subscript $a$ refers to wind axes and $W_x, W_y, W_h$ are the wind components with respect to the wind-axes.

Lift and drag are computed as follows,

$$\text{Lift} = \text{Lift}_{\text{wing}} + \text{Lift}_{\text{spoiler}}$$ \hspace{1cm} (37)

$$\text{Drag} = \text{Drag}_{\text{wing}} + \text{Drag}_{\text{spoiler}}$$ \hspace{1cm} (38)

$$\text{Lift}_{\text{wing}} = \frac{1}{2} \rho V_t^2 S_{\text{ref}} C_{L_{\text{wing}}}$$ \hspace{1cm} (39)

$$\text{Drag}_{\text{wing}} = \frac{1}{2} \rho V_t^2 S_{\text{ref}} C_{D_{\text{wing}}}$$ \hspace{1cm} (40)

In the foregoing, $S_{\text{ref}}$ is the wind reference obtained from BADA data. The drag coefficient is computed as a function of the lift coefficient; and the configuration of the aircraft (clean, approach, and landing). It can be written as follows,

$$C_{D_{\text{wing}}} = C_{D0,\text{BADA}}(configuration) + C_{D2,\text{BADA}}(configuration) C_{L_{\text{wing}}}^2$$ \hspace{1cm} (41)

$C_{D0,\text{BADA}}$ and $C_{D2,\text{BADA}}$ are the drag polar obtained from BADA.

The following nomenclature is relevant for the aerodynamic model of the spoiler.

$S_{\text{ref,spoiler}}$ \hspace{0.5cm} Spoiler reference area

$\theta$ \hspace{0.5cm} Spoiler deflection angle

The maximum drag resulting from the deployment of spoiler is computed as follows:

$$\text{Drag}_{\text{spoiler, max}} = \rho V_t^2 S_{\text{ref,spoiler}} C_{D_{\text{spoiler}}} \sin^2 \theta_{\text{max}}$$ \hspace{1cm} (42)

The spoiler deflection angle is computed as follows:

$$\theta = \sin^{-1} \left( \min \left\{ \frac{-\text{Drag}_{\text{spoiler}}, \text{Drag}_{\text{spoiler, max}}} {\rho V_t^2 S_{\text{ref,spoiler}} C_{D_{\text{spoiler}}}} \right\} \right)$$ \hspace{1cm} (43)

Notice the negative sign in front of $\text{Drag}_{\text{spoiler}}$, as it is expected that $\text{Drag}_{\text{spoiler}}$ is a negative quantity. The lift from spoiler deployment is computed as follows,

$$\text{Lift}_{\text{spoiler}} = -\frac{\rho V_t^2 S_{\text{ref,spoiler}} C_{L_{\text{spoiler}}}}{2} \sin 2\theta$$ \hspace{1cm} (44)

Again, notice the negative sign on the right hand side of the lift expression indicating negative lift.
Additional differential equations are used to simulate the dynamics associated with these controls. For example, the engine exhibits a lag in responding to commands, and similarly the aircraft takes some time to pitch up/down to realize the desired coefficient of lift. These lags and dynamics are modeled using the following differential equations:

\[
\begin{align*}
\dot{C}_L &= -k_{p,CL}(C_L - C_{com}) - k_{v,CL}\dot{C}_L \\
\dot{T} &= -k_T(T - T_{com}) \\
\dot{\phi} &= -k_{p,\phi}(\phi - \phi_{com}) - k_{v,\phi}\dot{\phi}
\end{align*}
\]

where \( C_{com}, T_{com}, \) and \( \phi_{com} \) are the guidance law commands, and the coefficients \( k_{CL} \) and \( k_T \) are designed depending on the dynamic response of the lift coefficient and engine thrust.

It should be noted that the reference trajectory synthesizer is based on a lesser fidelity model than the simulation model described above. As such the reference trajectory synthesizer does not account for the flight path angle \( \gamma_a \) dynamics; nor does it account for the lag in the thrust or the dynamics of roll and pitch axes. However, the guidance module is evaluated in a higher-fidelity simulation model to capture the effect of these modeling discrepancies. Figure 4 shows a block diagram of the closed-loop simulation environment.

![Block Diagram of the Closed-Loop Simulation Environment](image)

Figure 4. Block Diagram of the Closed-Loop Simulation Environment

V. Results

Closed-loop simulations obtained for different aircraft types flying along different routes at SFO and LAX are presented in the following sections. All results presented in this section are generated using the LNAV + VNAV guidance logic described in Section III and the closed-loop simulation environment described in Section IV.

A. A320 Along BIGSUR Route at SFO

Figure 5 shows the 3D trajectory of an A320 aircraft flying along the BIGSUR arrival route into San Francisco International Airport (SFO). Figure 6 shows the aircraft horizontal plane trajectory resulting from the use of LNAV guidance logic. Figure 7 and Figure 8 show the winds used in this simulation. They include both the reference trajectory winds, forecast winds, and the actual winds. The discrepancy between the reference trajectory winds and the actual winds poses a challenge for the LNAV and VNAV in tracking the trajectories.
The cross track errors shown in Figure 9 are mostly less than 0.1 nmi exceeding that value only during the final turn. Figure 10 shows the heading angle of the aircraft indicating two turns. Figure 11 shows the bank angle history required to execute the LNAV logic. The bank angle is constrained to be within ±30 degrees.

Figure 12 and Figure 13 show the flight level and airspeed histories as a function of the path length. The aircraft is seen descending from cruise flight level of 350 to the runway threshold. The speed is also seen reducing from the cruise speed setting to the landing speed setting of 135 knots. The altitude and speed tracking errors are shown in Figure 14 and Figure 15 respectively. The maximum altitude error occurs during the initial phase of the descent; it later settles down to a value less than 10 ft during the landing segment. The initial errors are due to the sharp transition from cruise to the constant Mach descent segment. The speed errors are less than 20 ft/s and assume a small value less than 10 ft/s during landing. The spikes seen in the speed error plots are due to the discontinuous speed changes and configuration changes during the landing segment. Future improvements will seek smoothing the reference trajectory to facilitate easier transition between these segments. Figure 16 shows the flight path angle which is mostly between 0 and -5 degrees as desired. Figure 17 shows the coefficient of lift required by the VNAV logic to track the vertical plane trajectory. Again the spikes are mostly attributed to the discontinuous segment changes. Figure 18 shows the thrust required by the speed-control logic. The thrust mostly stays close to the idle thrust during descent but strays from this setting during landing. The sudden and additional drag resulting from the configuration changes in the terminal airspace cause the departure of the thrust from the idle setting during the landing phase.
B. MD82 Along SHIVE Route at LAX

Figure 19-Figure 32 show plots for a MD82 flying along the SHIVE arrival route to LAX.
Figure 25. Bank Angle

Figure 26. Flight Level

Figure 27. Airspeed

Figure 28. Altitude Error

Figure 29. Airspeed Error

Figure 30. Flight Path Angle
C. TOA Uncertainty at Meterfix

The previous section presented results obtained from a single simulation. A Monte-Carlo simulation was conducted to evaluate performance metrics such as the TOA accuracy/error. Current sub-section describes the Monte Carlo simulation framework used to establish the TOA uncertainty for aircraft equipped with LNAV/VNAV capability and subject to wind uncertainties such as those shown in Figure 7 and Figure 8. In this study the Monte Carlo simulations are conducted from Freeze Horizon (about 120nmi away from the Meterfix) to Meterfix. Each Monte Carlo simulation simulates the uses the same aircraft, flying along the same route, using the same reference trajectory, experiencing the same deterministic wind forecast component, but a different random forecast error component. In this example the reference trajectory is chosen as the A320 aircraft flying along the BIGSUR route at SFO. The Meterfix is chosen as the BOLDR waypoint along the route. The trajectory is the same as the one presented in Section V A.

The outputs from the Monte-Carlo simulation are:

- Time-of-Arrival error = Actual TOA with LNAV/VNAV as a function of path length - Reference TOA as a function of path length (computed using the reference trajectory synthesis tool)
- \( \text{LNAV Errors} = \sqrt{\frac{\sum_{i=1}^{n} \Delta C_i^2}{n}} \), where \( \Delta C_i \) is the cross-track error at the \( i^{th} \) reference trajectory sample, and \( n \) is the total number of reference trajectory discretizations
- \( \text{VNAV Errors} = \sqrt{\frac{\sum_{i=1}^{n} \Delta h_i^2}{n}} \), where \( \Delta h_i \) is the altitude error at the \( i^{th} \) reference trajectory sample
- \( \text{Airspeed Errors} = \sqrt{\frac{\sum_{i=1}^{n} \Delta V_{ei}^2}{n}} \), where \( \Delta V_{ei} \) is the airspeed deviation from the reference trajectory at the \( i^{th} \) reference trajectory sample.

Figure 33 shows the TOA errors as a function of path length of the 500 Monte-Carlo trials. Figure 34 and Figure 35 show the mean and standard deviation of the errors as a function of path length. It can be inferred from Figure 35 that the uncertainty in TOA at the Meterfix for this aircraft example is 6.5 seconds. The initial path length equal to zero represents the Freeze Horizon and final path length close to 120nmi represents the Meterfix. Figure 36, Figure 37, and Figure 38 illustrate the distribution of VNAV, LNAV, and airspeed errors respectively. The histograms shown in these figures represent the distribution of LNAV, VNAV, and airspeed root-mean square errors for the 500 Monte-Carlo simulation trials.
Figure 33. TOA Errors Along Path

Figure 34. Mean of TOA Errors

Figure 35. STD of TOA Errors

Figure 36. Distribution of VNAV Errors

Figure 37. Distribution of LNAV Errors

Figure 38. Distribution of Airspeed Errors
D. TOA Uncertainty at Runway

The previous section presented Monte-Carlo simulation results from Freeze Horizon to Meterfix. In this section the Monte Carlo simulations are conducted from the Meterfix to the runway. Each Monte Carlo simulation simulates the uses the same aircraft, flying along the same route, using the same reference trajectory, experiencing the same deterministic wind forecast component, but a different random forecast error component. In this example the reference trajectory is chosen as the A320 aircraft flying along the BIGSUR route at SFO. The Meterfix is chosen as the BOLDR waypoint along the route and the runway is chosen as the 28R runway.

Figure 39 shows the TOA errors as a function of path length of the 500 Monte-Carlo trials. Figure 40 and Figure 41 show the mean and standard deviation of the errors as a function of path length. It can be inferred from Figure 35 that the uncertainty in TOA at the Runway for this aircraft example is 3.75 seconds. The initial path length equal to zero represents the Meterfix and final path length close to 30 nmi represents the Runway. Figure 42, Figure 43, and Figure 44 illustrate the distribution of VNAV, LNAV, and airspeed errors respectively.
Conclusions

This paper presents a design of 3D path tracking guidance laws to simulate the LNAV and VNAV capabilities of the FMS. Simulation results demonstrated the applicability of these guidance laws to different aircraft types and different arrival routes at different airports. The simulation results illustrate the performance of the proposed guidance law when subject to uncertainties in atmospheric forest. LNAV and VNAV technologies together constitute a key FMS capability that affect an aircraft’s time-of-arrival at points such as the Meterfix and the runway. The work estimates the time-of-arrival uncertainty associated with LNAV/VNAV using a Monte-Carlo simulation framework. Preliminary results indicate that the standard deviation of time-of-arrival errors at Meterfix to be 6.5 seconds and the standard deviation of the time-of-arrival errors at Runway to be 3.75 seconds. It should be noted that the above uncertainty only reflects the uncertainty resulting due to wind. Perfect knowledge of aircraft information such as weight, descent speeds, thrust, fuel consumption are assumed in generating these results. Further work is required to evaluate these uncertainties for different aircraft types, different forecast conditions, and routes. Characterization of this uncertainty is essential for estimating the benefits of equipping aircraft with LNAV/VNAV capability in the context of time-based scheduling. The uncertainty results generated in this paper are used in an accompanying paper to evaluate their beneficial impact on time-based scheduling.

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