4D Green Trajectory Design for Terminal Area Operations Using Nonlinear Optimization Techniques

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The work under this research deals with the development of a computational framework suitable for the design and analysis of 4D green trajectories for terminal airspace operations. First, a 4D-trajectory-based operational concept for terminal area operation consisting of ground-side automation and flight-deck-side automation is presented. The focus of the current paper is the development of 4D-trajectory design tools as part of the ground-side automation. The paper first identifies aircraft aerodynamic, fuel consumption, emissions, and noise models necessary for trajectory optimization based on open-source data such as the Base of Aircraft Data (BADA). A numerical trajectory optimization framework is then proposed for the design of 4D-trajectories. The framework is able to accommodate aircraft performance constraints, separation constraints, and airport capacity considerations, and it can model “green” considerations such as fuel & emissions minimization, and noise reduction. The trajectory optimization framework is demonstrated on single and multiple aircraft scenarios. Using parametric optimization approach the paper explores the relationship between the time-of-arrival at runway threshold and the fuel consumption for a B737 aircraft. In the multi-aircraft scenario the paper illustrates the implementation of 3 nmi separation criteria between a pair of aircraft. A companion paper deals with the flight-deck-side automation that tracks the 4D trajectory clearances created by the ground-side automation.

I. Introduction

ASA and the FAA have been involved in extensive efforts to develop advanced concepts, technologies, and procedures for the Next Generation Air Transportation System (NextGen)\textsuperscript{1-5}. The objective of these research efforts has been to improve the capacity, efficiency, and safety in the next-generation National Airspace System (NAS). Improvements can come in the form of more accurate and autonomous onboard navigational capabilities based on the Global Positioning System (GPS), more accurate surveillance capabilities such as Automatic Dependent Surveillance-Broadcast (ADS-B), advanced communication capabilities such as datalinks, improved vehicle designs, and finally improved air-traffic operations realized through advanced automation systems. A significant portion of the NextGen research is aimed at (i) developing ground-side automation systems to assist controllers in strategic planning operations, (ii) developing controller decision support tools to separate and space the traffic, and (iii) developing flight-deck-side automation to assist pilots in accomplishing airborne merging and spacing operations. The objective of the proposed research is to develop ground-side automation to enable 4D-Trajectory-Based Operations (4DTBO) in the terminal airspace. A computational framework for the design of 4D-Trajectories (4DTs) based on fundamental flight mechanics and nonlinear trajectory optimization techniques is developed and illustrated with sample scenarios in this paper.

Section II presents the findings from a literature survey of current and previous NextGen research on automation tools for terminal area operations. With the literature survey as a background, Section III motivates the benefits of and the requirements for 4DTBO and presents a 4DTBO concept for terminal airspace operations. Section IV details the modeling aspects of 4DT design. Section V formulates an optimization framework for the design of the 4DTs. Section VI presents trajectory optimization results obtained for singleaircraft and multipleaircraft scenarios.

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II. NextGen Terminal Area Operations Research

Automation concepts for NextGen terminal area operations can be categorized as (i) ground-side automation systems, or (ii) flight-deck-side automation systems. The research in the current paper is focused on ground-side automation. Reference 6 describes a concept for future high-density terminal air traffic operations that has been developed by the Airspace Super Density Operations (ASDO) researchers at NASA Ames Research Center. The concept described in Reference 6 includes five core capabilities: 1) Extended Terminal Area Routing, 2) Precision Scheduling Along Routes, 3) Merging and Spacing, 4) Tactical Separation, and 5) Off-Nominal Recovery. The first two capabilities are strategic planning tools and the remaining three are tactical decision support tools. In general tools developed for ground-side operations could be classified as (i) Strategic Planning Tools, and (ii) Tactical Decision Support Tools. The following sub-sections describe the past research under these two categories.

A. Strategic Planning Tools

Strategic planning tools deal with (i) route and runway assignment, and (ii) scheduling and sequencing at key points such as the meter fix and the runway threshold. The Traffic Management Advisor (TMA) is one of earliest scheduling tools developed at NASA Ames Research Center. The TMA supports en route controllers and managers with scheduling, spacing, and arrival flow management and is currently deployed at multiple Air Route Traffic Control Centers. The TMA uses a timeline graphical user interface (TGUI) to display schedule and sequence constraints at the traffic management position. Another tool is the Stochastic Terminal Area Scheduling Software (STASS), a massively parallel advanced scheduling software created at NASA Ames Research Center. STASS was designed as a time-based scheduling simulation tool that models aircraft arrivals in the terminal area. The scheduling is performed in two steps using (1) the Center Scheduler to schedule aircraft from the freeze horizon to the meter fixes and the runways and (2) the TRACON Scheduler to schedule aircraft to the runways upon their arrival at the meter fixes. Terminal Area Precision Scheduling and Spacing (TAPSS) is an integrated set of trajectory-based automation tools providing precision scheduling, sequencing, and controller merging and spacing functions. It is a strategic and tactical planning tool that provides Traffic Management Coordinators, En Route and Terminal Radar Approach Control air traffic controllers the ability to efficiently optimize the arrival capacity of a demand-impacted airport while simultaneously enabling fuel-efficient descent procedures. The TAPSS system consists of four-dimensional trajectory prediction, arrival runway balancing, aircraft separation constraint-based scheduling, traffic flow visualization, and trajectory-based advisories to assist controllers in efficient metering, sequencing, and spacing.

Among other research, Reference 10 proposed a decision support tool for high-density departure and arrival traffic management. Saraf et al. developed a dynamic scheduling algorithm using optimization techniques. Capozzi et al. developed a mixed-integer linear programming formulation for optimal routing and scheduling of Metroplex operations. In Reference 13 Saraf et al. compare different scheduling algorithms for Metroplex operations.

B. Tactical Decision Support Tools

Whereas strategic planning tools create schedules, tactical decision support tools help realize (i) schedule conformance and (ii) merging and spacing. The Final Approach Spacing Tool (FAST) is a decision support tool developed at NASA Ames Research Center for terminal-area air traffic controllers. The core capability of FAST, also known as "pFAST", where "p" stands for passive, generates runway assignments and landing sequences. An active version (aFAST) uses four-dimensional trajectory prediction algorithms to compute and display heading and speed advisories designed to sequence and space arrival aircraft at the runways. The aFAST advisories are expected to guide the controllers in realizing the schedule and the sequence created by pFAST. Controller Managed Spacing (CMS) refers to a set of controller decision support tools also built at NASA Ames Research Center. The CMS tools are: (i) timelines to provide a graphical depiction of the relationship between the Estimated Times of Arrival (ETAs) and Scheduled Times of Arrival (STAs) of aircraft crossing a specified location, (ii) Slot Marker Circles, which are a type of ghosting display that present the time-based schedule information spatially on the traffic display, and (iii) Speed Advisories that suggest air speeds that controllers could issue to correct schedule errors. A conflict detection and resolution system for terminal area operations, called Terminal-Tactical Separation Assisted Flight Environment (T-TSAFE), was developed for operational use in References 16 and 17. The 4D co-operative arrival manager (4D-CARMA) from DLR/EUROCONTROL provides control guidance (i.e., speed and heading) and timeline displays. The Dutch ANSP Luchtverkeersleiding Nederland (LVNL) developed the Speed and Route Advisor (SARA) automation system that is intended to enable controllers to guide arrival traffic into Amsterdam Schiphol.
Airport with improved arrival precision at the Initial Approach Fix (IAF). Researchers from MITRE developed decision support tools such as Merge Manager and Relative Position Indicator.  

### III. 4D Concept for Terminal Area Operations

The literature survey presented in the previous section conforms to the following paradigm of terminal area operations: (i) generate a strategic plan (schedule and sequence at few waypoints) for terminal operations using planning automation, and (ii) realize the plan using tactical decision support tools. The paradigm is compatible with the current-day Communication, Navigation, and Surveillance (CNS) infrastructure, especially the voice-based communication mechanism between the controllers and the flight crew. It does not place any new requirements on aircraft equipage. Thus, it offers a very realistic chance of transitioning these technologies to actual operations.

Research efforts originating from Europe have shown the benefits of trajectory optimization and the design of green trajectories. From a flight mechanics perspective, the 4DT defines various performance metrics such as the time of arrival, fuel consumption, and emissions.

A 4DT under the current research is defined as a "continuous" representation of three position coordinates with respect to time. Thus each 4DT can assume a very high-dimensional representation \((t,x,y,z)\). A Green 4DT can in addition specify an emissions an noise contract as well. Table 1 shows the components of a Green 4DT. It includes a 4DT contract the specifies the location of the aircraft at finely discreteized time steps. The Green Contracts specify the limits on emissions and noise as a function of the aircraft position.

#### Table 1. Green 4D Trajectory

<table>
<thead>
<tr>
<th>Time ((t))</th>
<th>4DT Contract</th>
<th>Green Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_0)</td>
<td>(\tau_{mf})</td>
<td>(CO_x_limit)</td>
</tr>
<tr>
<td>(\lambda_{mf})</td>
<td>(NO_{x}_limit)</td>
<td></td>
</tr>
<tr>
<td>(h_{mf})</td>
<td>(HC_limit)</td>
<td></td>
</tr>
<tr>
<td>(\tau_1)</td>
<td>(\lambda_1)</td>
<td>(Noise_limit)</td>
</tr>
<tr>
<td>(h_1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td></td>
</tr>
<tr>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td></td>
</tr>
<tr>
<td>(t_f)</td>
<td>(\tau_{rwy})</td>
<td></td>
</tr>
<tr>
<td>(\lambda_{rwy})</td>
<td>(\ldots)</td>
<td></td>
</tr>
<tr>
<td>(h_{rwy})</td>
<td>(\ldots)</td>
<td></td>
</tr>
</tbody>
</table>

The subscript \(mf\) refers to the beginning of the terminal area trajectory at the meterfix and the subscript \(rwy\) refers to the end of the terminal area trajectory at the runway threshold.

### A. Benefits of 4DTB0

4DToB0 for terminal area are expected to have the following benefits:

- Conventional scheduling tools typically compute and specify time of arrival to ensure separation at a few waypoints such as the meterfix and the runway threshold. 4DTs specify a time of arrival for every point along the route. Thus, 4DTs can plan for adequate separation all along the route as opposed to a few discrete points.

- Conventional controller advisories assume the form of route, speed, and altitude advisories, which have few degrees-of-freedom. 4DT clearances can offer better performance because each 4DT has infinite degrees of freedom.

- 4DTB0 facilitated by 4D-guidance systems on the flight-deck side can significantly reduce the controller workload. It is expected that the controller will have to make fewer tactical decisions.

- 4DTB0 facilitated by 4D-guidance systems on the flight-deck side significantly reduce the burden of ground-based trajectory prediction and can in fact improve the accuracy of ground-based trajectory prediction. Uncertainty associated with trajectory prediction is a major source of concern for several NextGen concepts/tools.

- 4DTB0 designed using trajectory-optimization tools offer the most comprehensive framework to design an integrated solution to routing, scheduling, and spacing aspects of terminal airspace operations.
• 4DTBO designed using trajectory-optimization tools enable the design of environmentally friendly “Green” trajectories by minimizing fuel, emissions, and noise.

• 4DTBO are ideal for combining multiple operational concepts such as Continuous Descent Arrivals (CDA)\textsuperscript{28}, Very Closely Spaced Parallel Runway Operations (VCSPRO)\textsuperscript{29}, and Collaborative Arrival Departure Operations Management (CADOM)\textsuperscript{30} over an entire metroplex.

B. Requirements of 4DTBO

While the benefits from using 4DTs are compelling, there are significant obstacles standing in the way of realizing 4D-trajectory-based operations. They are as follows:

• 4DTBO require reliable onboard navigation systems for tracking arbitrary 4DTs with high accuracy.

• 4DTBO require a data link capability for communicating 4DTs to the cockpit reliably.

• 4DTBO require sophisticated ground-side automation for the design of 4DT clearances.
  o 4DT clearances need to be computed in a computationally efficient manner to handle realistic terminal area traffic.
  o 4DT design automation cannot expect high-fidelity aircraft models which are protected intellectual property of aircraft manufacturers. It can only expect open source models such as the BADA. Some flight parameters such as aircraft weight can be expected to be communicated over the datalink.
  o 4DT design automation cannot expect highly accurate wind and temperature forecast. The accuracy of these forecasts is restricted by the accuracy of National Oceanic and Atmospheric Administration's (NOAA's) tools such as Rapid Update Cycle (RUC) or Rapid Refresh (RAP).
  o 4DT design should accommodate the controller's intervention to resolve off-nominal conditions.

• Finally, 4DTBO require 4D-Guidance Algorithms (4DGA) on flight-deck-side automation to track 4DT clearances issued by the ground-side automation.
  o 4DGA should work within the flight envelope constraints of the aircraft.
  o 4DGA should track accurately the 4DT clearance issued by the ground-side.
  o 4DGA should be robust to wind and temperature uncertainties.
  o 4DGA should be fuel-efficient.
  o 4DGA should not cause excessive wear and tear to the engines.

It is assumed that CNS infrastructure investments and equipment upgrades proposed under the Joint Planning and Development Office's (JPDO) Avionics Roadmap\textsuperscript{11} will result in GPS-based onboard navigation systems and data link capabilities in the far-term operational timeframe. These two technologies are expected to address the first two obstacles in the above list. Researchers from General Electric (GE)\textsuperscript{32} and MITRE\textsuperscript{33} have been conducting experiments with the downlink of 4DTs from the flight-deck. The objective of the proposed research is to address the ground-side automation and flight-deck-side automation necessary for 4DTBO.

C. Proposed 4DTBO Concept

Figure 1 shows a functional block diagram of the 4D-trajectory-based concept for terminal area operations. The concept involves both ground-side automation and flight-deck-side automation. The ground-side automation in turn consists of consists (i) 4D Green Trajectory Design Algorithms, (ii) conformance monitoring tools, (iii) conflict detection and resolution tools, and (iv) off-nominal recovery tools. The flight-deck-side automation consists of 4D Guidance Algorithms (4DGA). The ground-based automation designs 4D trajectories including emissions, noise contracts for individual aircraft taking into account the following considerations: (i) airport capacity, (ii) terminal airspace considerations, (iii) throughput considerations, (iv) safety considerations, (v) aircraft performance characteristics, and (vi) green considerations such as fuel, emissions, and noise. The ground-side automation also generates emissions and noise contracts for individual aircraft based on an aggregate goal. The ground-side
automation takes in as inputs real-time inputs such as surveillance data, atmospheric wind & temperature forecast, and the flight demand data to compute green 4D trajectories for all terminal area flights in an integrated manner taking into account all the above mentioned considerations. It should be noted that 4DTBO would still require controller decision support tools for conformance monitoring, CD&R, and off-nominal situation recovery. These tools are expected to handle off-nominal situations when one or more aircraft deviates from its 4DT. In such events the controller is expected to intervene if necessary using voice-based communications with the flight-crew.

The flight-deck automation resides in the Flight Management System (FMS) and generates the necessary guidance commands for tracking the 4DTs assigned by the ground-side automation while abiding by the emissions and noise contracts. The flight-deck-side automation takes in as inputs aircraft sensor measurements and generates throttle command, pitch attitude command, bank angle command, flap schedule, and gear schedule to realize the 4D trajectories.

Figure 1. Schematic of the 4DTBO for Terminal Area Operations

The focus of the current paper is on the design aspects of the green 4D trajectories, which is the ground-side component of this concept. A companion paper, Reference 34, deals with the flight-deck-side component of this concept.

D. 4D Trajectory Optimization Concept
The following are the terminal area operational considerations:

(i) Separation assurance,
(ii) Delay minimization,
(iii) Throughput maximization,
(iv) Fuel minimization,
(v) Emissions minimization,
These objectives and requirements can be translated into an optimization problem with 4D-trajectories as the decision variables:

\[
\begin{align*}
\text{min} & \sum_{i \in \mathcal{I}} (\alpha \ast \text{delay}_i + \beta \ast \text{fuel}_i) + \gamma t_{\text{final}} \\
\text{Subject to:} & \quad \text{Aircraft equations of motion} \\
& \quad \text{Aircraft Performance Constraints} \\
& \quad \text{Airspace Constraints} \\
& \quad \text{Separation Constraints} \\
& \quad \text{Landing Constraints} \\
& \quad \max(\text{noise}) \leq \text{noise}_{\text{threshold}} \\
& \quad \sum(\text{emissions}) \leq \text{emissions}_{\text{limit}}
\end{align*}
\]

The objective function penalizes the delay, the fuel consumption, and the landing time of the final aircraft, which is a measure of throughput. The first constraint is the dynamic propagation constraint to ensure the 4D trajectory solutions are feasible with respect to the equations of motion. The aircraft performance constraints enforce limits on aircraft performance such as stall speed and maximum load factor. Airspace constraints restrict the speed and the altitude of the aircraft at certain waypoints such as the metering fixes. Airspace requirements may also restrict the 3D routes (Standard Terminal Arrival Routes (STARs) and Departure Procedures (DPs)) along which the aircraft are nominally expected to travel. Separation constraints ensure that two aircraft are separated by the mandated horizontal (3 nmi) and vertical (1000 ft) separation standards. Landing constraints ensure that the terminal conditions of the 4D trajectory result in acceptable landing speeds and descent rates. Constraints involving noise and emissions are introduced to set upper limits on these variables. The model presented by above equation represents the grand optimization problem for entire terminal area flights. Current paper will demonstrate simpler scenarios using a single and multiple aircraft. Also, the current paper will focus on vertical plane scenarios as opposed to 3D scenarios. The above simplifications are purely for the purpose of demonstration and does not compromise on the ability of the approach to handle realistic terminal area traffic or 3D scenarios.

Section IV describes the modeling aspects of the optimization problem. Section V details the optimization framework for a single and multiple aircraft scenarios. Section VI presents the results for a single and multiple aircraft scenarios.

IV. Modeling

An aircraft model is essential for trajectory design. Whereas a higher-fidelity model is better for all design purposes, there are limitations in obtaining such models for the ground-side automation. Under the proposed concept, 4D trajectory design is done by the ground-based automation and 4D trajectory tracking is done by the flight-deck-based automation. The flight-deck-based automation is expected to be implemented by the Flight Management System (FMS) by aircraft manufacturers who have access to high-fidelity models. The following subsections describe the open-source models such as BADA used for trajectory design under the current research.

A. Aircraft Equations of Motion

Vertical-plane equations of motion are considered for the demonstrations in this paper.

\[
\begin{align*}
\dot{x} &= V \cos \gamma \\
\dot{h} &= V \sin \gamma \\
\dot{\gamma} &= (\frac{T \cos \alpha - D}{m}) - g \sin \gamma \\
\dot{\alpha} &= -\frac{g}{V} \cos \gamma + \frac{L}{mV} + \frac{Tsina}{mV} 
\end{align*}
\]

where \(x\) is the longitudinal position of the aircraft, \(h\) is the altitude of the aircraft, \(V\) is the airspeed of the aircraft, \(\gamma\) is the flight path angle, \(T\) is the thrust, \(L\) is the aerodynamic lift, \(D\) is the aerodynamic drag, \(m\) is the mass of the aircraft, \(g\) is the acceleration due to gravity, and \(\alpha\) is the angle of attack.

B. Aerodynamic Model

An aerodynamic model is necessary to compute the right-hand side of the equations of motion. Aerodynamic lift and drag computations are typically done using aerodynamic coefficients that are tabulated with respect to airspeed,
altitude, angle of attack, angle of sideslip, and all the control surface deflections. However, such a model may be expected to be available for trajectory design. A simplified model consisting of drag polars alone is considered in this research. Expressions for lift and drag are given below:

\[
\text{Lift:} \quad L = \frac{1}{2} \rho V^2 S_{\text{ref}} C_L \tag{3}
\]

\[
\text{Drag:} \quad D = \frac{1}{2} \rho V^2 S_{\text{ref}} C_D \tag{4}
\]

where \( \rho \) is the atmospheric density, \( S_{\text{ref}} \) is the reference area, \( C_L \) is the lift coefficient, and \( C_D \) is the drag coefficient. It is assumed that upper and lower limits on the lift coefficient, based on the flap setting and the gear setting, are available for the purpose of trajectory design. It is also assumed that the maximum speeds for the deployment of the flaps and gear are available to the trajectory design automation.

Limits on lift coefficient:
\[
C_{L,\text{min}}(\delta_{\text{flap}}, \delta_{\text{gear}}) \leq C_L(\delta_{\text{flap}}, \delta_{\text{gear}}) \leq C_{L,\text{max}}(\delta_{\text{flap}}, \delta_{\text{gear}}) \tag{5}
\]

Flap & Gear Deployment Speed Limits:
\[
V(\delta_{\text{flap}}) \leq V_{\text{max}}(\delta_{\text{flap}}) \\
V(\delta_{\text{gear}}) \leq V_{\text{max}}(\delta_{\text{gear}}) \tag{6}
\]

where \( \delta_{\text{flap}} \) and \( \delta_{\text{gear}} \) represent the flap and gear settings.

The drag coefficient is computed using the BADA drag polar coefficients.

\[
\text{Drag Coefficient and Drag:} \quad C_D = C_{D0}(\delta_{\text{flap}}, \delta_{\text{gear}}) + C_{D2}(\delta_{\text{flap}}, \delta_{\text{gear}}) C_L^2 \tag{7}
\]

where \( C_{D0} \) and \( C_{D2} \) are the drag polar coefficients. It should be noted that the drag polar coefficients are functions of the flap setting and the gear setting.

C. Fuel Model

A fuel consumption model of the following form is used for trajectory design:

\[
\text{Fuel:} \quad m_{\text{fuel}} = f_{\text{fuel}}(T, h, V) \tag{8}
\]

where \( T \) is the thrust, \( h \) is the altitude, and \( V \) is the airspeed. The fuel flow rate is computed from the thrust and the thrust specific fuel consumption (TSFC):

\[
\text{Fuel Flow Rate:} \quad \dot{m} = \text{TSFC} \times T \tag{9}
\]

where TSFC is in kg/s/N and \( T \) is thrust in Newtons (4.4482 N = 1 pound force).

The thrust-specific fuel consumption is given by:

\[
\text{Thrust-Specific Fuel Consumption:} \quad \text{TSFC} = C_{f1} \left(1 + \frac{V_{\text{TAS}}}{C_{f2}}\right) \tag{10}
\]

where the coefficients \( C_{f1} \) and \( C_{f2} \) are obtained from BADA.

D. Emissions Model

The emissions model is based on the International Civil Aviation Organization (ICAO) aircraft engine emissions database\(^{35}\). Fuel flow is used as the input and the Emissions Indices (EIs) for the three main pollutants are computed. \( \text{HC} \) stands for unburned hydrocarbons, \( \text{CO} \) stands for carbon monoxide, and \( \text{NO}_x \) stands for oxides of nitrogen. The ICAO data for the B737 engine type are shown in Figure 2. The EIs are computed using a table look-up.

![Figure 2. ICAO Engine Emissions Data](http://arc.aiaa.org/doi/abs/10.2514/6.2012-4755)
The Boeing Company devised an interpolation scheme using linear fits on a logarithmic scale, as shown in Figure 4. It is interesting to note that while the NO\textsubscript{x} emissions increase with increase in flow rate, the CO and HC emissions actually decrease. This method is believed to be more accurate than standard types of data interpolation, so this approach was used in the table look-up\textsuperscript{16}. The EIs are given in units of grams per kilogram of fuel, so multiplying by the fuel flow rate (with the appropriate unit conversion) gives mass flow rates for each of the pollutants. These mass flow rates can be integrated with time to get total mass values for a particular trajectory; the process is illustrated in Figure 3.

![Figure 3. Emissions Mass Computation](image)

![Figure 4. Emissions Index Interpolation](image)

E. Noise Model

The following noise model can be used for the trajectory design purpose:

\[
\text{noise} = f_{\text{noise}}(T, h, x, y, x_{\text{receiver}}, y_{\text{receiver}}) \tag{11}
\]

where \(x\) is the position of the aircraft, \(x_{\text{receiver}}\) and \(y_{\text{receiver}}\) are the horizontal-plane coordinates of the target location. A simplified computational model of noise was adopted for this study. The FAA’s Integrated Noise Model (INM) 37, soon to be superseded by the Aviation Environmental Design Tool (AEDT) is the standard airport noise analysis tool. However, there are some issues with this tool that prevented its use in this research. The two main drawbacks are the computation time involved in performing a noise analysis and the interface of the tool. Because it is desired to include noise constraints in the design of the trajectories, it is necessary to have a noise model that can...
be exercised repeatedly in the optimization algorithm. The INM would need to be set up to analyze only a few points for it to run fast enough to be practical in the optimization. Regardless of speed, the INM and AEDT are designed to be run interactively through a GUI, and there is no capability to call the tool as an external function from another software application. Another problem with the INM is the limitations on the data. The noise data contained in the INM include several thrust levels, but these do not include levels as low as was required for this study.

The first step in noise computation is to find the distance from the source to the receiver and the relative angle. Given \((x, y, z)_{\text{aircraft}}\) and \((x, y, z)_{\text{receiver}}\), the distance \(d\) and elevation angle \(\phi\) are computed as follows:

\[
\begin{align*}
    d &= \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \\
    \tan \phi &= \frac{\Delta z}{\sqrt{\Delta x^2 + \Delta y^2}} \\
    (\Delta x, \Delta y, \Delta z) &= (x, y, z)_{\text{aircraft}} - (x, y, z)_{\text{receiver}}
\end{align*}
\]

To get the relative azimuth angle, the \(x\) and \(y\) differences are transformed to a coordinate frame aligned with the aircraft’s heading \(\chi\).

\[
\begin{align*}
    \Delta x' &= \begin{bmatrix} \cos \chi & \sin \chi \\ -\sin \chi & \cos \chi \end{bmatrix} \Delta x \\
    \Delta y' &= \begin{bmatrix} \cos \chi & \sin \chi \\ -\sin \chi & \cos \chi \end{bmatrix} \Delta y
\end{align*}
\]

(13)

The relative angle \(\theta\) is then calculated from the transformed coordinates:

\[
\begin{align*}
    \tan \theta &= \frac{\Delta y'}{\Delta x'}
\end{align*}
\]

(14)

The source noise data are in the horizontal plane around the aircraft. No information is available about the dependence in the vertical plane, so it is assumed to be symmetric about the longitudinal axis of the aircraft.

The noise from the source \(S_{\text{source}}\) is obtained by table look-up as a function of thrust and relative angle.

\[
S_{\text{source}}(T, \theta) = S_{\text{source}}(T, \theta)
\]

(15)

The source data were obtained by running static run-up cases in the INM (Integrated Noise Model) (see Figure 5). Data were obtained for maximum sound level, which is independent of time, in contrast to average sound level or sound exposure level (SEL), the other common metrics. Cases were run for several different throttle settings, and a linear extrapolation was used below the minimum thrust level for which data were available. Sound levels at a radius of 300 feet from the source and at azimuth angles of 0 to 180 degrees were obtained (Figure 6), giving the reference levels and distance. This table of data is then interpolated for a given thrust level and receiver angle to get the source level, and then the propagation model is used to obtain the level at the receiver’s location.
The noise at the receiver $S_{receiver}$ is then computed as follows:

$$S_{receiver} = S_{source} + 20 \log \left( \frac{d_{ref}}{d} \right) - a_{ai}d - \Lambda(\phi)$$

(16)
Where \(d_{ref}\) is a reference distance, and \(a_{air}\) is an attenuation constant, and \(\Lambda\) is an adjustment for air-to-ground attenuation, which is a function of the elevation angle:

\[
\Lambda = 1.137 - 0.0229\phi + 9.72e^{-0.142\phi}
\]  
(17)

It should be noted that there are a variety of factors that affect sound propagation. There is a dependence on the frequency of the sound, for instance, that was not included in this model. The model still provides a way of capturing the overall effects of variations in the trajectory, though.

When there are multiple sources of noise such as multiple aircraft in the terminal area, the total sound level can be obtained as follows:

\[
L_{\text{total}} = 10 \cdot \log \left( \sum_{i=1}^{n} 10^{L_i/10} \right)
\]  
(18)

where \(L_i\) is the individual noise contribution at the receiver and \(L_{\text{total}}\) is the cumulative value.

V. 4D Trajectory Optimization Framework

A. Problem Data

As a first step towards generating the 4D trajectories for multiple aircraft, it is necessary to create individual aircraft trajectories satisfying the terminal airspace constraints, the landing constraints, and the aircraft performance constraints. Terminal airspace typically extends to about 35 nmi from the runway threshold and 10000 ft altitude, and aircraft typically enter the terminal airspace with a calibrated airspeed speed of 250 knots. Whereas the terminal airspace definition identifies the set of initial conditions for the 4D trajectories, the landing constraints determine the final-time conditions. In addition to the entry and landing conditions, each aircraft has to abide by its performance constraints while transitioning from the entry point to the runway threshold. A B737 aircraft is considered for the single aircraft trajectory design demonstration. The problem data for the 4D trajectory design consists of the following initial conditions for a B737 aircraft:

\[
x_{\text{initial}} = -35 \text{ nmi}, h_{\text{initial}} = 10000 \text{ ft} \\
V_{\text{initial}} = 289 \text{ knots}
\]  
(19)

Upper and lower limits of lift coefficients for different flap settings:

\[
C_{L_{\text{min}}} \leq C_L(\delta_{\text{flap}} = 0) \leq C_{L_{\text{max}}}
\]

\[
C_{L_{\text{min}2}} \leq C_L(\delta_{\text{flap}} = 5) \leq C_{L_{\text{max}2}}
\]

\[
C_{L_{\text{min}3}} \leq C_L(\delta_{\text{flap}} = 15, \delta_{\text{gear}} = 1) \leq C_{L_{\text{max}3}}
\]

\[
C_{L_{\text{min}4}} \leq C_L(\delta_{\text{flap}} = 30, \delta_{\text{gear}} = 1) \leq C_{L_{\text{max}4}}
\]  
(20)

Upper limit on airspeed for individual flap deployments for B757:

\[
V(\delta_{\text{flap}} = 5) \leq 225 \text{ knots}
\]

\[
V(\delta_{\text{flap}} = 15) \leq 195 \text{ knots}
\]

\[
V(\delta_{\text{flap}} = 30) \leq 185 \text{ knots}
\]  
(21)

And finally the wind forecast,

\[
W = W_{\text{RUC}}(x, h, t)
\]

\[
W = W_{\text{RUC}}(h)
\]  
(22)

The objective of the 4D trajectory design framework is to create a trajectory that transfers the aircraft from the terminal airspace initial conditions to the desired landing conditions while satisfying the aircraft performance constraints. A direct numerical trajectory optimization framework will be used for trajectory design. The first step in this approach involves the selection of decision variables for the optimization problem.

B. Decision Variables

In this research, the airspeed, flight path angle, thrust, and lift coefficient at discrete time instances are chosen as the optimization decision variables. It should be noted that the optimizer is free to not only choose the values of the
airspeed, flight path angle, thrust, and lift coefficient at the discrete time instance but also to choose the times associated with them. In this work, it is also assumed that the user specifies a final time or landing time. A total of 100 decision variables are used for a single aircraft as shown below:

| Decision Variables: | \[ \{t_1, V_{1,1}, T_1, C_{L_1}\}, \{t_2, V_{2,2}, T_2, C_{L_2}\}, \ldots, \{t_{20}, V_{20,20}, T_{20}, C_{L_{20}}\} \] (23) |

C. Objective Function
The first step in solving an optimization problem is to satisfy the constraints. Therefore, no objective function is chosen at this time. Candidate objective functions that are of interest in this research are (i) final time and (ii) total fuel.

| Objective Function: | none \[ (\text{Current}) \] (24) |
| Objective function for minimum time problem: | \( t_{20} \) \[ (\text{Future}) \] (25) |
| Objective function for minimum fuel problem: | \( m_{\text{fuel}} = \int_0^{t_{20}} m_{\text{fuel}} (V_{L_{20}}, T_{L_{20}}) \, dt \) \[ (\text{Future}) \] (26) |

D. Constraints
The flap and gear settings are not directly modeled as decision variables. An indirect approach is used which is shown below:

| Flap & Gear Settings | \( \delta_{\text{flap}}(t_{11}, t_{13}) = 0^\circ \) 
| | \( \delta_{\text{flap}}(t_{11}, t_{13}) = 5^\circ \) 
| | \( \delta_{\text{flap}}(t_{14}, t_{16}) = 15^\circ \) 
| | \( \delta_{\text{flap}}(t_{17}, t_{20}) = 30^\circ \) 
| | \( \delta_{\text{gear}}(t_{11}, t_{13}) = 0, \delta_{\text{gear}}(t_{17}, t_{20}) = 1 \) \[ (\text{Future}) \] (27) |

The decision variable \( t_{20} \) is constrained to be equal to the desired final time. The times associated with the 20 discretizations of the decision variables need to be monotonically increasing, which results in the following constraints:

| Time Constraints: | \( t_1 < t_2 < \cdots < t_{20} \) 
| | \( t_{20} = t_f \) \[ (\text{Future}) \] (28) |

The decision variables \( V_1 \) and \( V_{20} \) are set to the initial airspeed and the desired landing speed, respectively. The flap deployment speed constraints are imposed on the speed variables \( V_{11,13} \), which represent the speeds associated with 5° flap deployment. Similarly, the speed restrictions for 15° and 30° flap deployments are enforced on decision variables \( V_{14,16} \) and \( V_{17,20} \), respectively. The speed values used here are for the B737 model.

| Speed Constraints: | \( V_1 = V_{\text{initial}} \) 
| | \( V_{\text{final}} \leq V_{1,20} \leq V_{\text{initial}} \) 
| | \( V_{\text{final}} \leq V_{11,13} \leq 225 \text{ knots} \) 
| | \( V_{\text{final}} \leq V_{14,16} \leq 195 \text{ knots} \) 
| | \( V_{\text{final}} \leq V_{17,20} \leq 185 \text{ knots} \) 
| | \( V_{20} = 134 \text{ knots} \) \[ (\text{Future}) \] (29) |

Flights are typically flying with a flight path angle of approximately -3° (i.e., on a 3° glide slope) on the final approach. It is also unreasonable to expect flights to climb on their descent segments. Even though 20 flight path angle decision variables are used in the optimization problem, frequent changes could affect passenger comfort. Therefore, the flight path angle is held constant over multiple discretization time instances. Together, these two requirements can be modeled using the following inequality constraints:

| Flight Path Angle Constraints: | \(-3^\circ \leq \gamma_{1,20} \leq 0 \) 
| | \( \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 \) 
| | \( \gamma_6 = \gamma_7 = \gamma_8 = \gamma_9 = \gamma_{10} \) 
| | \( \gamma_{11} = \gamma_{12} = \gamma_{13} \) \[ (\text{Future}) \] (30) |
The lift coefficient of an aircraft is influenced weakly by Mach number and altitude; and strongly by angle of attack and flap settings. The minimum and maximum lift coefficient for each of the four flight segments – no flaps, 5° flaps, 15° flaps, and 30° flaps – are modeled using the following inequality constraints on the decision variables $C_{l_{1:4}}$.

\[
\begin{align*}
C_{l_{min1}} & \leq C_{l_{1:4}} & \leq C_{l_{max1}} \\
C_{l_{min2}} & \leq C_{l_{1:4}} & \leq C_{l_{max2}} \\
C_{l_{min3}} & \leq C_{l_{1:4}} & \leq C_{l_{max3}} \\
C_{l_{min4}} & \leq C_{l_{1:4}} & \leq C_{l_{max4}}
\end{align*}
\] (31)

Aircraft typically fly close to their idle thrust till the flaps are deployed and then make changes to thrust upon deployments of flaps. In the trajectory optimization process, the idle thrust is computed for $\delta_{idle\text{-}throttle} \pm 5$ and is used to define the upper and lower limits on the thrust decision variables $T_{1:20}$.

\[
\begin{align*}
T_{1\min} &= T(\delta_{idle\text{-}throttle} - 5, h_0, V_0) \\
T_{1\max} &= T(\delta_{idle\text{-}throttle} + 5, h_0, V_0) \\
T_{20\min} &= T(\delta_{idle\text{-}throttle} - 5, h_r, V_f) \\
T_{20\max} &= T(\delta_{idle\text{-}throttle} + 5, h_r, V_f) \\
T_{j\min} &= T_{1\min} + \frac{j}{20} (T_{20\min} - T_{1\min}) \\
T_{j\max} &= T_{1\max} + \frac{j}{20} (T_{20\max} - T_{1\max}) \\
T_{j\min} &\leq T_j \leq T_{j\max} \quad j = 1..20
\end{align*}
\] (32)

The landing speed is modeled by setting the $V_{20}$ decision variable, which in this case is set equal to 134 knots, the landing speed of a B737. To model the position and the altitude associated with the landing time, the origin for the coordinate system is chosen at the runway threshold. Therefore, the landing constraints result in the final value of $x$ being equal to zero and the final of $h$ equal to 50 ft at the final time. These pose another set of constraints.

\[
x_{final} = 0, \quad h_{final} = 50 \text{ ft}
\] (33)

The model for evaluating the constraints using the decision variables is given below. The wind speeds obtained from the RUC forecast are used to convert the airspeed to inertial speed before propagating the kinematic equations of motion. Thus, the 4D trajectory design process explicitly accounts for the nominal wind forecast data. The airspeed and flight path angle are linearly interpolated in between the discrete time specifications $t_{1:20}$. The propagation time step $\Delta t$ is chosen as 1 second.

\[
\begin{align*}
W[k] &= W_{RUC}(h[k]) \\
V_{\text{iner}}[k] &= \sqrt{(V[k] \cos(y[k]) + W[k])^2 + (V[k] \sin(y[k])}^2 \\
x[k + 1] &= x[k] + V_{\text{iner}}[k] \cos(y[k]) \Delta t, k = 1..(N - 1), N = \frac{t_f}{\Delta t} \\
h[k + 1] &= h[k] + V_{\text{iner}}[k] \sin(y[k]) \Delta t, k = 1..(N - 1)
\end{align*}
\] (34)

\[
x_{final} = x[N] = 0 \\
h_{final} = h[N] = 50 \text{ feet}
\] (35)

In addition to satisfying the kinematic Equations of Motion (EOM), the airspeed, flight path angle, thrust, and lift coefficient time histories also need to satisfy the EOM for airspeed and flight path angle given by Eq. 2. The Left Hand Side (LHS) of these differential equations is discretized with a 1-second time step as shown below:

\[
\begin{align*}
V_{LHS}[k] &= \frac{V[k + 1] - V[k]}{\Delta t} \\
\gamma_{LHS}[k] &= \frac{\gamma[k + 1] - \gamma[k]}{\Delta t}
\end{align*}
\] (36)

The aerodynamic quantities of interest to evaluate the Right Hand Side (RHS) of the EOM are evaluated using the standard atmospheric model and the BADA drag polars as follows:

\[
\begin{align*}
\rho[k] &= \rho_{\text{STADMO}}(h[k]) \\
L[k] &= \frac{\rho[k] V^2[k] S_{ref} c_L[k]}{2}
\end{align*}
\] (37)
The discretized representation of the RHS of the EOM is shown below.

\[ D[k] = \frac{1}{2} \rho[k] V^2[k] S_{ref} C_D[k] \]

The EOM RHS is given by:

\[ V_{\text{RHS}}[k] = \frac{T[k] \cos \theta[k] - D[k] - g \sin \gamma[k]}{m} \]

\[ \dot{\gamma}_{\text{RHS}}[k] = -\frac{g}{V[k]} \cos \gamma[k] + \frac{L[k]}{m V[k]} + \frac{T[k] \sin \alpha[k]}{m V[k]} \]  

Ideally, the LHS and the RHS have to match exactly, but to facilitate the convergence of the optimization process, the norm of the difference between the LHS and RHS is upper bounded by small values \( \varepsilon_V \) and \( \varepsilon_\gamma \) as shown below:

\[ \sum_{k=0}^{N} (\dot{V}_{\text{LHS}}[k] - \dot{V}_{\text{RHS}}[k])^2 \leq \varepsilon_V \]

\[ \sum_{k=0}^{N} (\dot{\gamma}_{\text{LHS}}[k] - \dot{\gamma}_{\text{RHS}}[k])^2 \leq \varepsilon_\gamma \]  

Other outputs of interest such as the fuel consumption, emissions, and noise are computed as follows:

\[ m_{\text{fuel}}[k] = \int_{0}^{t_f} m_{\text{fuel}}(T[k], h[k], V[k]) \, dt \]

\[ m_{\text{emissions}} = \int_{0}^{t_f} m_{\text{emissions}} \, dt \]

\[ SNL[k] = f_{\text{noise}}(x[k], h[k], T[k], x_{\text{receiver}}, y_{\text{receiver}}) \]

### VI. Results

#### A. Single Aircraft 4D Trajectory Design

The optimization problem was solved using MATLAB’s `fmincon` function. Figure 7 shows the design airspeed and design inertial speed of the aircraft obtained from the numerical optimization. The final airspeed is seen to be equal to the 134 knots landing requirement. The inertial speed is less than the airspeed because of the headwind experienced by this aircraft, as shown in Figure 8 and Figure 9. Figure 10 shows the flight path angle time history, which lies between 0° and −3° throughout as stipulated.

The thrust profile required by the aircraft to achieve the airspeed and flight path angle profile is shown in Figure 11. The thrust peaks towards the end because of the increase in drag resulting from the deployment of the flaps. The time history of the other decision, the lift coefficient, is shown in Figure 12. Both the thrust and coefficient of lift are seen to stay within limits.

The flap schedule is shown in Figure 13. It can be inferred from Figure 14 that the airspeed restrictions (in the case of the TSRV model, the maximum speed restrictions for 5°, 15°, and 30° flap settings are 225 knots, 195 knots, and 185 knots, respectively) for the deployment of the flaps are satisfied by this solution.

Figure 17 and Figure 18 show the resulting \( x \) and \( h \) time histories. It should be noted that the terminal values of both these variables are satisfied as specified by their landing constraints. Interestingly, the altitude time history in this case results in a continuous descent.

The lift-to-weight ratio shown in Figure 21 is very close to 1 and varies between 96% and 103%. Figure 19 and Figure 20 show the \( \dot{V} \) and \( \dot{\gamma} \) profiles respectively. The figures show both the LHS and RHS of the \( \dot{V} \) and \( \dot{\gamma} \) dynamics equations. It can be seen that the \( \dot{V}_{\text{LHS}} \) matches closely with \( \dot{V}_{\text{RHS}} \). The flight path angle is mostly constant over each segment, resulting in \( \dot{\gamma}_{\text{LHS}} = 0 \) for the most part. It should be noted that, while \( \dot{\gamma}_{\text{RHS}} \) seems different from \( \dot{\gamma}_{\text{LHS}} \), it is a small variation over a reference that is mostly zero.
Figure 7. Airspeed Time History
Figure 8. Head Wind Time History
Figure 9. Head Wind With Respect to Altitude
Figure 10. Flight Path Angle Time History
Figure 11. Required Thrust Time History

Figure 12. Lift Coefficient Time History

Figure 13. Required Flap Schedule

Figure 14. Flap Schedule With Respect to Airspeed
Figure 15. Required Gear Schedule

Figure 16. Gear Schedule w. r. t. Airspeed

Figure 17. X Position Time History

Figure 18. Altitude Time History
Figure 19. Airspeed Dynamics Equation LHS and RHS

Figure 20. Flight Path Angle Dynamics Equation LHS and RHS
The 4DT clearance for this aircraft would be all the information in the $t \ vs \ x$ and $t \ vs \ h$ plots in Figure 17 and Figure 18 respectively. At one second time discretization this could result in transmission of a matrix of size $(610 \times 3)$ of the type "double". Reference 34 demonstrates tracking of this 4DT clearance to a very high accuracy using flight-deck-side automation that consists 4D-guidance algorithms. The demonstration is done on a high-fidelity TSRV simulation model and simulates wind and temperature uncertainties.

B. Time of Arrival Vs. Fuel Consumption

The results in the previous section are for an aircraft trying to meet the final time conditions for a specified final time. It is desirable to keep the final time small, or in other words, arrive as early as possible. However, it is also of interest to evaluate the impact of arriving sooner or later on the fuel consumption. Towards realizing this objective, the trajectory optimization was carried out for different final times. Figure 22 shows the fuel consumption variation with respect to the final time. Interestingly, there seems to be an optimal final time for minimum fuel consumption. Arriving later than the optimal final time only increases the fuel consumption. It should be noted that the current trajectory model does not use a path-stretch to realize the given final time. Instead, it only uses speed variations. Another interesting point to note is that the fuel consumption behavior does not change even when the high-fidelity model is used as seen in Figure 23. The high-fidelity model shown in Figure 23 is the TSRV simulation model for a B737 aircraft, the Transport Systems Research Vehicle, obtained from NASA Langley Research Center. The optimal final time is the same for both the BADA-fuel-model-based fuel computation and TSRV-fuel-model-based fuel computation. Therefore, if the ground-based automation identifies the optimal final time using the BADA fuel model, there is a good chance that the same number would be obtained by the flight-deck-based automation. In other words it is possible for the ground-side automation to determine the fuel optimal trajectory for the aircraft. But fuel-optimality is only one aspect. It is important that the aircraft be separated from other aircraft as well. The automation in this case could choose the final time that results in deconfliction with other traffic and issue that 4D trajectory as a clearance.
Figure 22. Total Fuel Consumed Vs. Final Time

Figure 23. Comparison of BADA and TSRV Fuel Consumption Models
C. Multi-Aircraft Trajectory Design

The previous sections dealt with the design of 4D trajectories for a single aircraft, while satisfying the desired landing conditions and the aircraft performance constraints. Separation assurance for terminal area operations requires generating 4D trajectories for multiple aircraft that satisfy the 3 nmi separation constraint. The 3 nmi separation requirement couples the two trajectory optimization problems and results in a higher-order optimization problem. For a two-aircraft scenario, this results in 200 decision variables, 100 for each aircraft. Each aircraft is individually required to satisfy all the constraints modeled in Section D. In addition, the aircraft are also expected to satisfy the 3 nmi separation constraint at all times. The separation constraint is modeled as follows:

\[
x_{ac1}[k], k = 1..N_t, N_t = \frac{t_{f1}}{\Delta t}
\]

\[
x_{ac2}[k], k = 1..N_t, N_t = \frac{t_{f2}}{\Delta t}
\]

Separation Requirement as Inequality Constraints:

\[
\Delta x = x_{ac2}[k] - x_{ac1}[k] < -3\text{nmi}, k = 1..N_t
\]

The results obtained from the coupled trajectory optimization are shown in Figure 24 – Figure 27. The separation constraint of 3 nmi can be seen to be satisfied throughout in Figure 24 and assumes a minimum value at the last time step, when the landing aircraft crosses the threshold. A separation value larger than 3 nmi can be included in the optimization framework as a buffer. It should be noted both the leading and the trailing aircraft individually satisfy the landing conditions as seen in Figure 25 – Figure 27.

**Figure 24. Longitudinal Separation Between the Two Aircraft**
Figure 25. Longitudinal Position Coordinate Time Histories

Figure 26. Altitude Time Histories
The 4DT clearance for this aircraft would be all the information in the \( t \ vs \ x \) and \( t \ vs \ h \) plots in Figure 25 and Figure 26 respectively. At one second time discretization this could result in transmission of two matrices of size \((610 \times 3)\) of the type "double". Reference 34 demonstrates tracking of these two 4DT clearances to a very high accuracy using flight-deck-side automation that consists 4D-guidance algorithms. The demonstration consists of two high-fidelity TSRV simulation models and simulates wind and temperature uncertainties. It is shown that the two aircraft tracking their individual 4DT clearances also assures separation at all times.

**VII. Conclusion**

The paper presented a unique 4D-trajectory-based operations concept for terminal area operations. The concept consists of both ground-side automation and flight-deck-side automation. The following are the key features of this concept:

1. Unlike conventional scheduling tools that specific time-of-arrival at key waypoints the current concept generates a complete 4D-trajectory and green contracts for individual aircraft.
2. The paper lays the foundation to a very rigorous nonlinear trajectory optimization approach for designing 4D green trajectories suitable for terminal area operations.
3. 4D trajectories generated using this approach are assured of separation all along the routes.
4. The 4D-trajectory design process is based completely on open-source models.

The approach is tested on a single and multiple aircraft trajectory scenarios and is seen to result in realistic trajectories. Single aircraft trajectory optimization studies reveal an interesting tradeoff between the time of arrival and fuel consumption. It is observed there is an optimal time of arrival for minimum fuel consumption. Arriving faster than that time or later than that time both result in fuel consumption, and delays in the case of arriving later. Multiple aircraft trajectory optimization demonstrated the ability to accommodate separation into the 4D-trajectory design process. Future work shall involve design of these trajectories for multiple aircraft in realistic terminal area scenarios using green performance metrics such as emissions and noise.
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