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AIR TRAFFIC FLOW MODELING, ANALYSIS AND CONTROL

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ABSTRACT

Development of a methodology for mathematical modeling and analysis of air traffic flow is presented. The modeling approach describes the air traffic environment in terms of traffic counts in user-defined elements of the airspace system, and traffic flows between these elements. The resulting Eulerian model of air traffic allows dynamic analysis and flow-control system design using well-established control theoretic approaches. The primary advantage of the Eulerian approach is that the dimension of the air traffic flow model depends only on the number of defined elements in the traffic network, and remains invariant with respect to the number of aircraft in the airspace system. Analytical development, computational implementation, and traffic flow modeling and control of a regional airspace are discussed in the paper.

1. INTRODUCTION

Several research initiatives are currently underway[1,2] within NASA and the FAA to develop mathematical models of air traffic flow, and to utilize them for the development of analysis methodologies that will lead to more efficient management of the U.S. National Airspace System (NAS). Air traffic simulation and analysis tools such as the NASA-developed Future ATM Concepts Evaluation Tool[3] (FACET) and its derivatives[4] can simulate the trajectories of every aircraft in the NAS, using flight plan data, winds, and aero-propulsive models of each aircraft. While such detailed models are very useful for modeling, analyzing, and controlling traffic at the individual aircraft level, it may be difficult to apply analytical methodologies to them for higher-level applications such as managing the flow of traffic streams.

An alternate modeling methodology was recently advanced in References 5 and 6. In that work, an Eulerian approach[7] was developed to model the air traffic flow environment. The Eulerian air traffic flow approach models the airspace in terms of line elements approximating airways, together with merge and diverge nodes. Since the Eulerian modeling technique spatially aggregates the air traffic, the order of the airspace model depends only on the number of line elements used to represent the airways, and not on the number of aircraft operating in the airspace. The development of the modeling technique discussed in References 5 and 6 is based on prior research on road traffic modeling,[8-12] and is termed as the Eulerian Traffic Flow (EATF) Model.

This paper extends the work described in References 5 and 6 by modeling the airspace in terms of latitude-longitude surface elements. In the present EATF modeling approach, the air traffic environment is modeled in terms of eight-connected, two-dimensional surface elements, instead of the one-dimensional line elements employed in the work of References 5 and 6. Instead of using airways to describe the system dynamics as in the previous work, the traffic flow patterns in each surface element are directly computed using the FACET software. The size of the latitude-longitude tessellation is chosen based on maximum expected aircraft speed in the NAS.

It was shown in References 5 and 6 that if the speed of air traffic can be considered to be nearly constant within each line element, the Eulerian modeling approach yields a set of linear, time-varying, discrete-time difference equations. Linearity allows models to be derived directly from the geometric definition of the latitude-longitude tessellation, and larger models can be assembled from linear models of smaller dimensions using linear algebraic operations. The dynamic relationships between any pairs of variables can be determined using computational linear algebra.[13] Section 2 will describe the formulation of the EATF model.

References 5 and 6 demonstrated that the EATF model can be used to develop real-time decision aids to predict the stability and robustness of the air traffic environment. Moreover, it was shown in References 5 and 6 that the EATF model can be used to synthesize flow control schemes. Linear multivariable control theory[14] provides a variety of design methodologies that can be used with the

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EATF model to derive stable and robust flow control algorithms. These algorithms can either be used as decision aids for manual control, or possibly for automatic flow control. Section 3 will describe a model predictive control algorithm to achieve desired flow rates through specified regions of the airspace.

In addition to being useful for synthesizing flow control algorithms, the EATF model can be used to derive a variety of analytical results. For instance, the reachability of the airspace with respect to flow metering and ground stop controls at various locations in the airspace can be assessed using well-conditioned numerical algorithms. Such analyses can be valuable for improving the efficiency of flow control operations. Section 4 will illustrate the computation of the reachable set for a region of the airspace. Conclusions will be presented in Section 5.

2. EULERIAN TRAFFIC FLOW MODELING USING EIGHT-CONNECTED SURFACE ELEMENTS

For modeling purposes, the NAS is divided horizontally into latitude-longitude grids with user-specified dimensions. Each of these grid elements is called a SEL (Surface ELement). The Eulerian model is built up by assembling models of individual SELs. One-degree latitude-longitude increments can be used for coarse modeling, while smaller grid size may be more appropriate for modeling finer details. Figure 1 shows a typical latitude-longitude tessellation used to model the U.S. airspace. The present flow modeling effort focuses on Class A airspace; hence the SELs model aircraft flying at or above 18,000 ft.

The numbering convention of the surface elements (i,j) is that j is increasing from left to right and i is increasing from bottom to top (increasing longitude and latitude). In a given surface element, it is assumed that there are eight streams representing the traffic flows in the north, northeast, east, southeast, south, southwest, west, and northwest directions. Consequently, each element has eight inputs and nine outputs, to and from the eight streams in each of the adjacent SELs. The arrival stream is the ninth output of the SEL. For element (i,j), the adjacent SELs are (i-1,j-1), (i-1,j), (i-1,j+1), (i,j-1), (i,j+1), (i+1,j-1), (i+1,j), and (i+1,j+1), as shown in Figure 2.

If an element contains one or more airports, there will be departure and arrival flows, $q^{depart}_{(i,j)}$, $q^{arrive}_{(i,j)}$ added to or subtracted from each stream. In addition, if an element lies on the boundary of the airspace being modeled, there will be additional inputs $q^{exo}_{(i,j)}$ due to traffic entering the airspace from elsewhere, such as international flights. The various inputs for a typical stream are shown in Figure 3. The naming convention for the eight streams in a SEL is indicated in Figure 4, where x represents the number of aircraft in a stream at any given time. In this model the system variables are defined only at discrete points in time, so the number of aircraft in a stream as a function of time is $x(k\tau)$, where τ is the specified time step, and k = 0, 1, 2, … , or for brevity, simply $x(k)$.

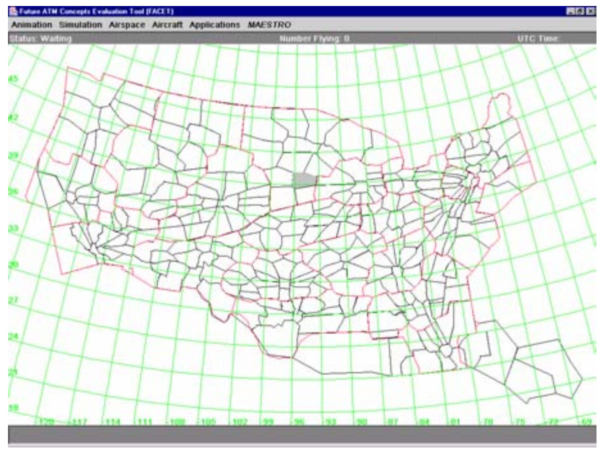

Figure 1. Latitude-Longitude Tessellation Used in Eulerian Flow modeling

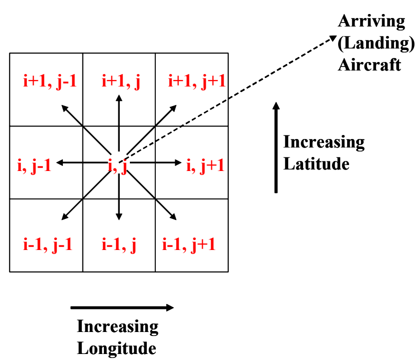

Figure 2. Surface Element Numbering Convention

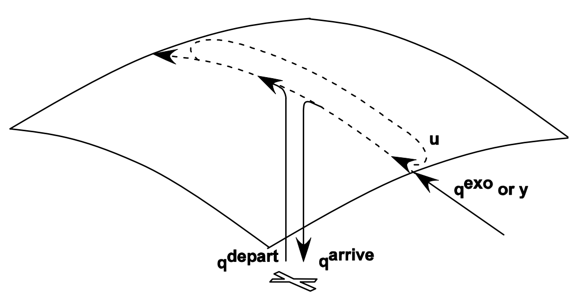

Figure 3. Inputs to a Surface Element

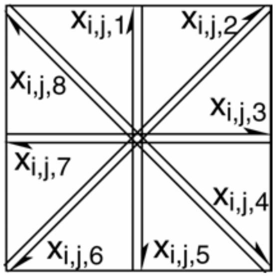

Figure 4. Nomenclature of Surface Element State Variables

Discretizing the aircraft flow into eight directions may require aircraft to switch between different streams as they transit through a SEL. These changes are captured in the EATF model through the flow divergence parameters (β). A percentage β_{mn} of the flow entering the SEL in stream n exits through another stream m, or lands (q^{arrive}). Since an aircraft in any stream may exit by any of the 8 streams, or by landing, for a given SEL there is a matrix of 9×8 = 72 flow divergence parameters. In order to satisfy the principle of conservation of aircraft in a surface element, for each input flow n, the divergence parameters to all of the outputs must add up to unity, i.e.;

$$\sum_{m=1}^{9} \beta_{mn} = 1$$

Note that one of the β_{mn} is not independent. By convention, let

$$\beta_{nn} = 1 - \sum_{\substack{m=1 \\ m \neq n}}^{9} \beta_{mn}$$

It is assumed that an aircraft will nominally remain in the same stream, so the *default* values are:

$$\beta_{mn} = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases}$$

Even when divergences occur, it is highly unlikely that an aircraft will reverse direction in the same element, so certain parameters can reasonably be assumed to be zero. In general, most of the elements of the matrix will be zero. The parameters β_{mn} for each element (i,j) are determined from FACET trajectory propagation data.

As in References 5 and 6, the behavior of the state variables x in each stream of a surface element can be described by summing input and output flows. Some aircraft might not exit a given element within the time step, so this is accounted for in the dynamics with the inertia parameter $a_{(i,j,p)}$. The parameter $a_{(i,j,p)}$ is a ratio that represents the aircraft that do not exit steam p in the surface element (i,j) within the time step τ, due to differences in element sizes or differences in speed. They are determined from FACET trajectory propagation data; the default values are zero when no aircraft are present.

The control variables in the EATF model $u_{(i,j,p)}$ are the aircraft flow rates held back in a SEL through flow control actions and the departure traffic flow rates from the airports in the SEL $q^{depart}_{(i,j,p)}$.

For surface element (i,j), the dynamics of the streams can be represented by the difference equations:

$$x_{(i,j,1)}(k+1) = a_{(i,j,1)} \sum_{m=1}^{8} \beta_{(i,j,1,m)} x_{(i,j,m)}(k)$$
$$+ \tau u_{(i,j,1)}(k) + \tau y_{(i-1,j,1)}(k) + \tau q^{depart}_{(i,j,1)}(k) + \tau q^{exo}_{(i,j,1)}(k)$$
$$x_{(i,j,2)}(k+1) = a_{(i,j,2)} \sum_{m=1}^{8} \beta_{(i,j,2,m)} x_{(i,j,m)}(k)$$
$$+ \tau u_{(i,j,1)}(k) + \tau y_{(i-1,j-1,2)}(k) + \tau q^{depart}_{(i,j,2)}(k) + \tau q^{exo}_{(i,j,2)}(k)$$
$$x_{(i,j,3)}(k+1) = a_{(i,j,3)} \sum_{m=1}^{8} \beta_{(i,j,3,m)} x_{(i,j,m)}(k)$$
$$+ \tau u_{(i,j,3)}(k) + \tau y_{(i,j-1,3)}(k) + \tau q^{depart}_{(i,j,3)}(k) + \tau q^{exo}_{(i,j,3)}(k)$$
$$x_{(i,j,4)}(k+1) = a_{(i,j,4)} \sum_{m=1}^{8} \beta_{(i,j,4,m)} x_{(i,j,m)}(k)$$
$$+ \tau u_{(i,j,4)}(k) + \tau y_{(i+1,j-1,4)}(k) + \tau q^{depart}_{(i,j,4)}(k) + \tau q^{exo}_{(i,j,4)}(k)$$
$$x_{(i,j,5)}(k+1) = a_{(i,j,5)} \sum_{m=1}^{8} \beta_{(i,j,5,m)} x_{(i,j,m)}(k)$$
$$+ \tau u_{(i,j,5)}(k) + \tau y_{(i+1,j,5)}(k) + \tau q^{depart}_{(i,j,5)}(k) + \tau q^{exo}_{(i,j,5)}(k)$$
$$x_{(i,j,6)}(k+1) = a_{(i,j,6)} \sum_{m=1}^{8} \beta_{(i,j,6,m)} x_{(i,j,m)}(k)$$
$$+ \tau u_{(i,j,6)}(k) + \tau y_{(i+1,j+1,6)}(k) + \tau q^{depart}_{(i,j,6)}(k) + \tau q^{exo}_{(i,j,6)}(k)$$
$$x_{(i,j,7)}(k+1) = a_{(i,j,7)} \sum_{m=1}^{8} \beta_{(i,j,7,m)} x_{(i,j,m)}(k)$$
$$+ \tau u_{(i,j,7)}(k) + \tau y_{(i,j+1,7)}(k) + \tau q^{depart}_{(i,j,7)}(k) + \tau q^{exo}_{(i,j,7)}(k)$$
$$x_{(i,j,8)}(k+1) = a_{(i,j,8)} \sum_{m=1}^{8} \beta_{(i,j,8,m)} x_{(i,j,m)}(k)$$
$$+ \tau u_{(i,j,8)}(k) + \tau y_{(i-1,j+1,8)}(k) + \tau q^{depart}_{(i,j,8)}(k) + \tau q^{exo}_{(i,j,8)}(k)$$

where y is the output flow rate of a stream in an adjacent surface element, and u represents a possible metering control imposed by an air traffic flow controller. The parameters $a_{(i,j,p)}$ and β_{mn} depend on time, or equivalently the index k.

These equations represent the most general form of the EATF model. Normally, most streams will have no metering control, and many surface elements will not have any airports. While the external inputs q^{exo} are shown in all eight directions, an element can have at most five external inputs since it is assumed that at least one edge of the element is always connected to the rest of the airspace being modeled. Moreover, for a given stream, the inflows y and q^{exo} are mutually exclusive because the external inflow q^{exo} replaces the inflow y for a stream that comes from outside the airspace being modeled.

It should also be noted that the departures q^{depart} may not be exactly as shown above. Because the present modeling considers only en route traffic (at or above 18,000 ft), an aircraft may enter the flow of traffic in a SEL adjacent to (or even away from) the SEL containing the airport from which the flight departed. Also, every stream around an airport may not have a departure term, since departure traffic is often constrained to fly along certain directions within the terminal area. The same caveats apply to arriving aircraft. It has also been observed that for some airports, departing aircraft going in the same direction may enter the traffic flow in either the element containing the airport or a neighboring element. This may be a result of a difference in climb rates among different aircraft types.

The stream outputs for a surface element (i,j) can be defined as:

$$y_{(i,j,m)}(k) = \left(1 - a_{(i,j,m)}\right) \sum_{n=1}^{8} \beta_{(i,j,m,n)} x_{(i,j,n)}(k) - u_{(i,j,m)}(k), \quad m = 1,2,\ldots,8$$

The arrival traffic leaving that surface element can be represented by the additional output:

$$y_{(i,j,m)}(k) = \sum_{n=1}^{8} \beta_{(i,j,m,n)} x_{(i,j,n)}(k), \quad m = 9$$

Note that no control is permitted to act directly on the arrival flights, since the objective is to meter traffic before it gets to the terminal area.

Denoting:

$$x(k+1) = \begin{bmatrix} x_{(i,j,1)}(k+1) \\ x_{(i,j,2)}(k+1) \\ \vdots \\ x_{(i,j,8)}(k+1) \end{bmatrix}, \quad x(k) = \begin{bmatrix} x_{(i,j,1)}(k) \\ x_{(i,j,2)}(k) \\ \vdots \\ x_{(i,j,8)}(k) \end{bmatrix},$$

$$u(k) = \begin{bmatrix} u_{(i,j,1)}(k) \\ u_{(i,j,2)}(k) \\ \vdots \\ u_{(i,j,8)}(k) \end{bmatrix}$$

$$q^{depart}(k) = \begin{bmatrix} q^{depart}_{(i,j,1)}(k) \\ q^{depart}_{(i,j,2)}(k) \\ \vdots \\ q^{depart}_{(i,j,8)}(k) \end{bmatrix}, \quad q^{arrive}(k) = \begin{bmatrix} q^{arrive}_{(i,j,1)}(k) \\ q^{arrive}_{(i,j,2)}(k) \\ \vdots \\ q^{arrive}_{(i,j,8)}(k) \end{bmatrix},$$

$$q^{exo}(k) = \begin{bmatrix} q^{exo}_{(i,j,1)}(k) \\ q^{exo}_{(i,j,2)}(k) \\ \vdots \\ q^{exo}_{(i,j,8)}(k) \end{bmatrix}$$

The system dynamics equation can be expressed in a compact form as:

$$x(k+1) = A(k)x(k) + Bu(k) + B_d q^{depart}(k) + B_e q^{exo}(k)$$

The departure traffic may be subdivided according to those airports where they will be controlled (by a ground delay) and those where will not. It is assumed that external traffic q^{exo} cannot be controlled directly. If the controlled inputs are combined into a vector $v(k)$, and all other inputs are combined into a disturbance vector $w(k)$, the dynamic equation for a surface element becomes

$$x(k+1) = A(k)x(k) + B_1 v(k) + B_2 w(k)$$

The state vector $x(k)$ can be initialized using traffic data and then propagated forward in time. These equations can be used to facilitate analysis and synthesis of flow control strategies.

Typically, not all states are of interest for analysis and flow control. The output equation can be formulated to provide the output vector of interest as:

$$y(k) = C(k)x(k) + D_1 v(k)$$

The EATF model consists of the time-varying difference equation for the state vector, and the time-varying algebraic equation for the output vector. These equations can be formulated for surface elements in any desired region of the NAS, and combined together to form a basis for analysis and flow-control system design.

2.1. MAESTRO: Software for EATF Modeling

Since it is tedious to determine the coefficients of the EATF model manually, an automatic modeling technique has been developed using the FACET software as the foundation. The trajectory predictions generated by FACET are used to compute the coefficients of the EATF model, and to assemble models of multiple surface elements to form composite models of the NAS. The modeling technique has been implemented in the form of a software package called MAESTRO (Modeling and Analysis Environment for Studying Traffic–flow Requirements and Operations).

The FACET graphical user interface has been augmented to allow interactive Eulerian modeling and analysis. In addition to automatic modeling features, the software provides several traffic flow analysis functions. Current functionality of MAESTRO includes:

1. Graphical EATF modeling of the U.S. national airspace at user-specified resolution.
2. Analysis Tools
   - Controllability analysis with respect to selected metering and departure control locations.
   - Reachable set computations.
   - Latency analysis between selected origin-destination pairs.
   - Stochastic analysis of air traffic flows.
   - Stability and robustness analysis with respect to specified flow control strategies.
   - Model decomposition for decentralized control.
3. Synthesis tools
   - Model predictive air traffic flow control.

Other features of the software package are currently being developed. These would be described in a future publication. The following section describes the use of the EATF model generated from the MAESTRO software for air traffic flow control.

3. MODEL PREDICTIVE CONTROL OF AIR TRAFFIC FLOW USING EATF MODEL

As discussed earlier in this paper, one of the important uses of the EATF model is to enable the determination of flow control strategies. Flow control strategies can be used to compute metering speeds at various user-specified locations, and/or departure rates from specific airports in order to maintain desired traffic densities in one or more regions of the airspace, or arrival flow rates at specified destination airports. In terms of the EATF model, the control problem can be symbolically posed as follows.

Given the Eulerian traffic flow model:

$$x(k+1) = A(k)x(k) + B_1(k)u(k) + B_2(k)w(k)$$

$$y(k) = C(k)x(k) + D_1(k)u(k)$$

find a sequence of controls $u(k)$ that achieves the control objectives

$$y(k) \le y_{Desired}(k), \quad x(k) \le x_{Desired}(k)$$

while minimizing the usage of controls:

$$\min_u \sum_{k=1}^{m} u(k).$$

Control variables are subject to the constraints:

$$0 \le D_1(k)\, u(k) \le C(k)x(k)$$

The performance index expresses the requirement that the flow control commands be as few or as small as possible. The control constraints enforce the physical limitation that outflow rate from a surface element cannot be negative, and that it must be less than or equal to the number of aircraft naturally exiting the surface element. The latter condition prevents the flow control strategies from requiring the acceleration of aircraft in a surface element to meet flow control objectives.

Although it is feasible to apply classical control system design methods such as pole placement and LQR theory[14] to this problem, much of their effectiveness is lost when control constraints are imposed on the solution. Moreover, since constraint satisfaction is essential for the practical implementation of the flow control strategies, it is desirable to use a control technique that can explicitly incorporate the control constraints in the solution process.

Model Predictive Control[15] (MPC) is one such technique. The MPC technique first uses the EATF model to predict the influence of flow control commands on aircraft flows at locations of interest in the airspace. It then uses an optimization scheme to compute the control settings that will yield the desired flow characteristics. A flowchart of the Model Predictive Control methodology for air traffic flow control using the EATF model is illustrated in Figure 5. This flow chart illustrates the computations that must be carried out at each sample.

The central element of MPC is a predictor. In the most general case, the predictor is a numerical simulation model of the dynamic system. However, the linearity of the EATF model allows the derivation of analytical predictors, as illustrated below.

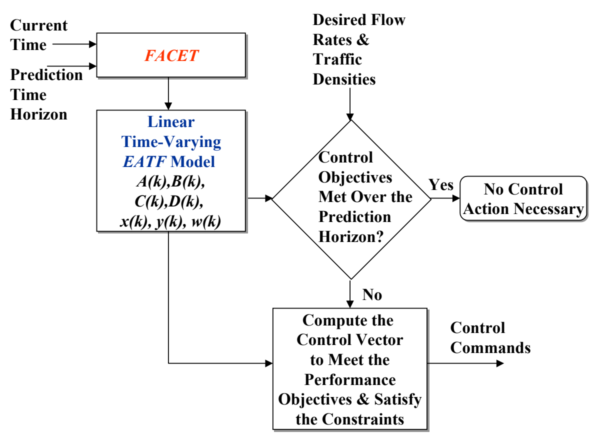

Figure 5. Model Predictive Air Traffic Flow Control Using the EATF Model

A multi-step predictor can be set up by propagating an initial condition vector $x(1)$ through the EATF model over multiple time steps as follows.

$$x(2) = A(1)\, x(1) + B_1(1)\, u(1) + B_2(1)\, w(1)$$

$$y(1) = C(1)x(1) + D(1)u(1)$$

$$x(3) = A(2)\,x(2) + B_1(2)\,u(2) + B_2(2)\,w(2)$$

$$y(2) = C(2)x(2) + D(2)u(2)$$

Eliminating the state $x(2)$ from these equations yields:

$$x(3) = A(2)\,A(1)x(1) + A(2)\,B_1(1)u(1) + A(2)\,B_2(1)\,w(1) + B_1(2)\,u(2) + B_2(2)\,w(2)$$

$$y(2) = C(2)A(1)\,x(1) + C(2)B_1(1)\,u(1) + C(2)B_2(1)\,w(1) + D(2)u(2)$$

Similarly,

$$x(4) = A(3)\,x(3) + B_1(3)\,u(3) + B_2(3)\,w(3)$$

$$y(3) = C(3)x(3) + D(3)u(3)$$

Eliminating $x(3)$ from these equations yields:

$$x(4) = A(3)\,A(2)A(1)x(1) + A(3)\,A(2)B_1(1)\,u(1) + A(3)\,A(2)B_2(1)\,w(1) + A(3)B_1(2)\,u(2) + A(3)B_2(2)\,w(2) + B_1(3)\,u(3) + B_2(3)\,w(3)$$

$$y(3) = C(3)A(2)\,A(1)x(1) + C(3)A(2)\,B_1(1)\,u(1) + C(3)A(2)\,B_2(1)\,w(1) + C(3)B_1(2)\,u(2) + C(3)B_2(2)\,w(2) + D(3)\,u(2)$$

Equations such as these can be derived for any number of sample steps. They can then be assembled in the form of a matrix equation relating the system outputs to the initial conditions, disturbances and control inputs to yield:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ \vdots \end{bmatrix} = \begin{bmatrix} C(1) \\ C(2)A(1) \\ C(3)A(2)A(1) \\ C(4)A(3)A(2)A(1) \\ \vdots \end{bmatrix} x(1)$$

$$+ \begin{bmatrix} D(1) & 0 & 0 & 0 & \cdots \\ C(2)B_1(1) & D(2) & 0 & 0 & \cdots \\ C(3)A(2)B_1(1) & C(3)B_1(2) & D(3) & 0 & \cdots \\ C(4)A(3)A(2)B_1(1) & C(4)A(3)B_1(2) & C(4)B_1(3) & D(4) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} u(1) \\ u(2) \\ u(3) \\ u(4) \\ \vdots \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots \\ C(2)B_2(1) & 0 & 0 & 0 & \cdots \\ C(3)A(2)B_2(1) & C(3)B_2(2) & 0 & 0 & \cdots \\ C(4)A(3)A(2)B_2(1) & C(4)A(3)B_2(2) & C(4)B_2(3) & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} w(1) \\ w(2) \\ w(3) \\ w(4) \\ \vdots \end{bmatrix}$$

The foregoing multi-step predictor can be expressed in a more compact form as:

$$Y = \widetilde{A}\,x(1) + \widetilde{B}_1\,U + \widetilde{B}_2\,W$$

The MPC problem can then be set up using this predictor.

Given the current value of the state vector $x(1)$, the disturbance vector W, and the matrices $\widetilde{A}, \widetilde{B}_1, \widetilde{B}_2$, the control problem is to find the vector U that minimizes the control effort:

$$J = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & \cdots \end{bmatrix} U$$

subject to the inequality constraints:

$$Y \le Y_{Desired}\text{ , or }\quad \widetilde{A}\,x(1) + \widetilde{B}_1\,U + \widetilde{B}_2\,W \le Y_{Desired}\text{ ,}$$

$$0 \le U \le U_{Max}$$

The designer has the responsibility for selecting the prediction time horizon. As a variation of the foregoing problem statement, one can consider different weighting for different components of the control vector in the performance index.

It may be observed from the foregoing discussion that the problem of finding the control vector that minimizes the objective function while satisfying the constraints is a linear programming problem that can be solved using well-known numerical methods such as the Simplex algorithm.[16] For the present work, a MATLAB®[17]-based linear programming solver LIPSOL[18] was used to generate results.

The Dallas / Fort Worth Air Route Traffic Control Center (ZFW), for the time period between 1200 and 1400 local time on a typical day, was chosen to demonstrate the model predictive flow control methodology. The control objective is to maintain a specified flow rate into a defined area surrounding the DFW airport, as shown in Figure 6. Each surface element in the model covers 1 degree latitude and 1 degree longitude, or approximately 60 nm north-south by 50 nm east-west. For an aircraft cruise speed of 400 knots, this spatial discretization leads to a sample time step of 6 minutes. The EATF model of this region contains 36 surface elements, which results in 288 state variables. For simplicity, the flows into DFW airport are summed by quadrant (NW, NE, SE, SW), resulting in four model outputs to be controlled. A total of 40 metering controls were next introduced, based on observing the traffic flow over the specified time period. The hourglass shaped metering symbols are shown in the appropriate surface elements in Figure 6. Short line segments on the metering symbols indicate the directions (streams) in which the metering controls are applied.

The flow characteristics along the four DFW arrival directions with no flow control (open loop) are shown in Figure 7. This figure shows the nature of the traffic flow into the DFW area in the absence of any flow control actions.

The Model Predictive Control (MPC) strategy is next implemented, with the objective of maintaining flow in each stream at or below 2 aircraft per time step. Note that this requires substantial reduction in flows, particularly along the NE, NW and SE directions. The performance of the model predictive closed-loop flow control system is first illustrated using a linear time-varying simulation. Linear, time-varying models constructed by running the FACET and MAESTRO software for the 2-hour duration is used to set up this simulation. Figure 8 shows the closed-loop flow control system response in the linear time-varying simulation. Since the coefficients of the linear, time-varying models are real numbers, rounding of inputs and outputs causes some deviation from the commanded values. Otherwise, the command tracking is achieved to within 1 aircraft in each sample.

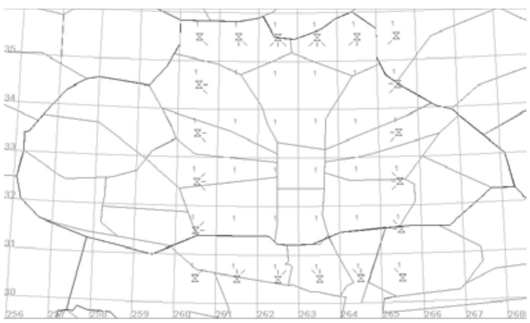

Figure 6. Dallas / Fort Worth Center with Metering Controls

Next, the flow control commands generated by the model predictive control algorithm are implemented in FACET. Figure 9 shows the closed-loop system performance. In order to generate these results, MAESTRO-generated linear, time-varying models from the current time-step to the 2-hour prediction window are used to compute the flow control commands. From Figure 9, it may be observed that the flow control is not as precise as that from the linear, time-varying simulation given in Figure 8, but is close to it. Except at isolated sample instants, the flow control system is able to deliver command tracking to within 2 aircraft per time step. Since the linear, time-varying EATF models involve spatial and temporal discretization of the aircraft trajectories, it may not be possible to achieve exact control of air traffic flow. However, the closed-loop behavior represents a considerable improvement over the open-loop time histories. It may be possible to obtain tighter flow control by reducing the discretization intervals, and by introducing metering controls in additional surface elements. Approaches for tightening the flow control loop are currently being pursued.

4. REACHABLE SET COMPUTATION USING THE EATF MODEL

Reachable set defines the range of outputs that can be achieved using admissible controls. The information provided by reachable sets can help assess the degree of flow control that can be accomplished using the defined control variables. Reachable sets are defined for a time interval of interest. The linear predictor model employed in the predictive control technique can be readily adapted for computing the reachable set.

The objective is to determine the range of outputs that can be achieved by varying the controls between the permissible minimum and maximum values. A variant of the linear programming algorithm can be used to compute the reachable set in the present problem. Since these computations only examine the boundary of the reachable set, it assumes that every point within the set is reachable, so that the reachable set does not contain any 'holes.' In other words, it is expected that the reachable set does not have the characteristics of Swiss cheese.

As an example, reachable set for the time interval between 1212 and 1236 local time in the region of the DFW airport along three flow directions is illustrated in Figure 10. The reachable set is in the form of a polyhedron. Figure 10 indicates, for example, that under the existing traffic conditions, the SE quadrant flow rate can be controlled between 2 and 6 aircraft per time step. Although it is feasible to construct reachable sets of arbitrary dimension, it sophisticated graphics packages will be required to visualize them.

5. CONCLUSIONS

This paper discussed the development of the Eulerian air traffic flow modeling methodology, and its use in analysis and control of air traffic flow. Unlike the traditional trajectory modeling of individual aircraft, the Eulerian approach focuses on representing the dynamic characteristics of elements of airspace and its relationship to the neighboring elements. Consequently, the order of the Eulerian model depends only on the number of elements being modeled and not on the number of aircraft. It was shown in previous research that the Eulerian air traffic flow modeling results produces linear, time-varying models which can be analyzed using linear algebraic techniques.

The present work extends previously developed Eulerian air traffic models by representing the airspace using eight-connected surface elements. Consequently, each surface element is represented using eight state variables and nine outputs. A computer software package, called MAESTRO, has been developed to automatically generate Eulerian models from the definition of the airspace of interest and spatial discretization intervals. Model predictive control methodology for achieving flow control in regions of interest was then demonstrated using the Eulerian model. Finally, the computation of reachable set was illustrated. Reachable sets are useful for assessing whether a specific control objective is achievable. Future research will discuss the use of the Eulerian air traffic flow model for conducting analysis of the airspace.

ACKNOWLEDGEMENT

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REFERENCES

1. *Proceedings of the First Aviation System Technology Advanced Research (AvSTAR) Workshop*, NASA/CP-2001-209616, NASA Ames Research Center, September 21-22, 2000.
2. *Proceedings of the AvSTAR Workshop*, NASA Ames Research Center, March 13-15, 2001.
3. Bilimoria, K.D., Sridhar, B., Chatterji, G., Sheth, K.S., and Grabbe, S., "FACET: Future ATM Concepts Evaluation Tool," *Air Traffic Control Quarterly*, Vol. 9, No. 1, March 2001, pp. 1–20.
4. Menon, P. K., Crawford, L. S., and Lam, T., "Software Environment for Investigating Decentralized ATM Concepts," Final report submitted under NASA SBIR contract No. NAS2-00042, NASA Ames Research Center, June 7, 2000.
5. Menon, P. K., Sweriduk, G. D., and Bilimoria, K. D., "A New Approach for Modeling, Analysis and Control of Air Traffic Flow," *AIAA Guidance, Navigation and Control Conference*, August 5 – 8, 2002, Monterey, CA.
6. Menon, P. K., and Sweriduk, G. D., "Methods for Assessing the Stability and Robustness of Distributed ATM Systems," Final Report Prepared under NASA SBIR contract No.NAS2-01024, Optimal Synthesis Report No. OSI-NASA-0102, Los Altos, CA, August 9, 2001.
7. Prandtl, L. and Tietjens, O. G., *Fundamentals of Hydro- and Aeromechanics*, Dover, New York, NY, 1957.
8. Lighthill, M. J., and Whitham J. B., "On Kinetic Waves I: Flow Movement in long rivers; II A Theory of Traffic Flow on Long Crowded Roads, *Proceedings of the Royal Society*, Part A, 229, 1955, pp. 281-345.
9. Richards, P. I., "Shockwaves on the Highway", *Operations Research*, Vol. 4, 1956, pp.42-51.
10. Newell, G. F., "A Simplified Theory of Kinematic Waves in Highway Traffic, Part I: General Theory", *Transportation Research*, Vol. 27B, No. 4, 1993, pp.281-287.
11. Daganzo, C. F., "The Cell Transmission Model: A Dynamic Representation of Highway Traffic Consistent with The Hydrodynamic Theory", *Transportation Research*, Vol. 28B, No. 4, 1994, pp.269-287.
12. Daganzo, C. F., "The Cell Transmission Model, Part II: Network Traffic", *Transportation Research*, Vol. 29B, No. 2, 1995, pp.79-93.
13. Anderson, F., et al, *LAPACK User's Guide*, Society for Industrial and Applied Mathematics(*SIAM*), Philadelphia, PA, August 1999. (http://www.netlib.org/lapack).
14. Ogata, K., *Discrete-Time Control Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1987.
15. Levine, W. S.(Editor), *The Control Handbook*, CRC Press, Boca Raton, FL, 1996.
16. Kreyszig, E., *Advanced Engineering Mathematics*, Wiley, New York, NY, 1993.
17. Anon, *MATLAB User's Manual*, The MathWorks, Inc., Natick, MA, 1998.
18. Zhang, Y., "User's Guide to LIPSOL," Dept. of Computational and Applied Mathematics, Rice University, Houston, Texas, February, 1998.

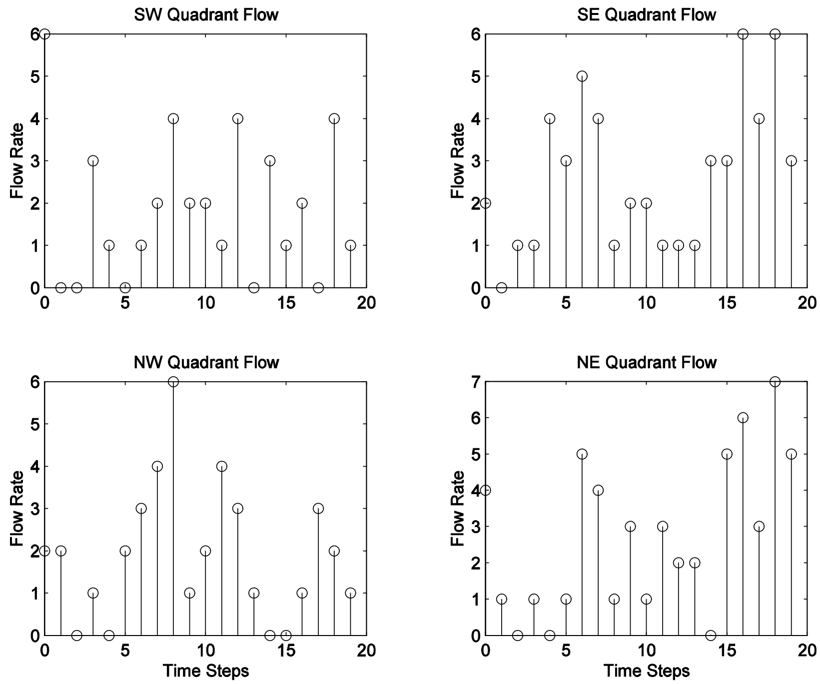

Figure 7. Open-Loop Flows for DFW Area on a Typical Day between 1200 and 1400 local time

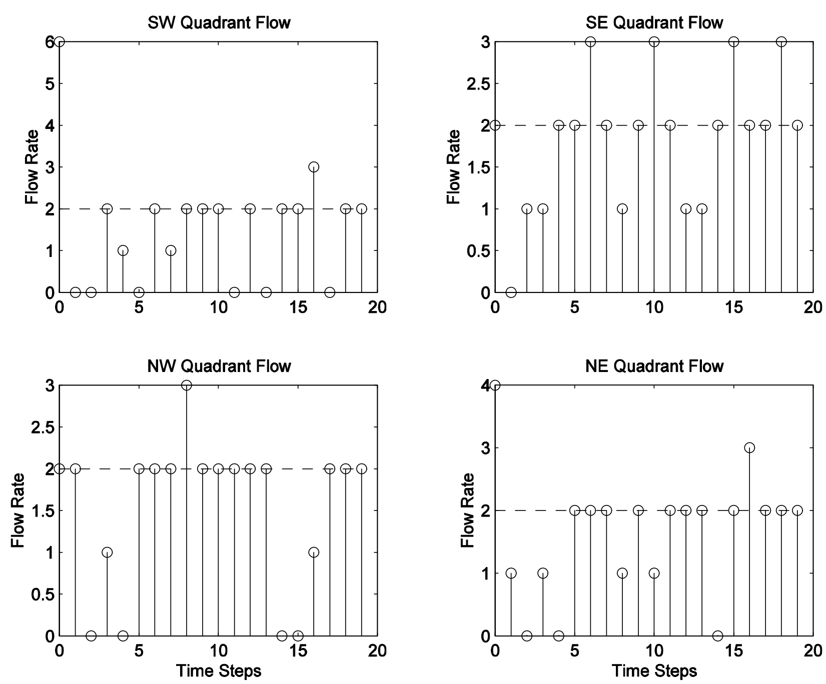

Figure 8. Closed-Loop Flow (with Metering Control) for DFW Area – Linear Time-Varying Simulation

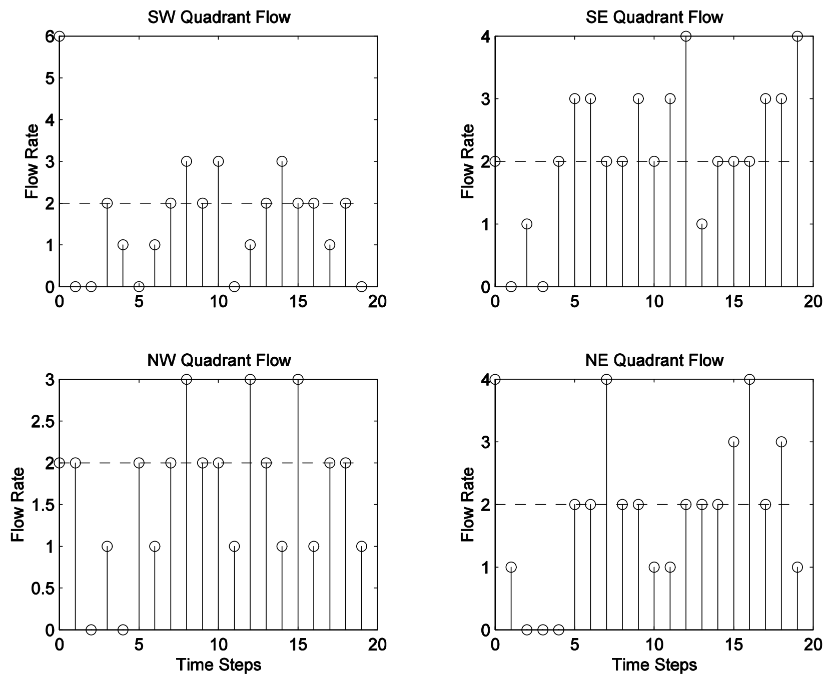

Figure 9. Closed-Loop Flow (with Metering Control) for DFW Area – FACET Simulation

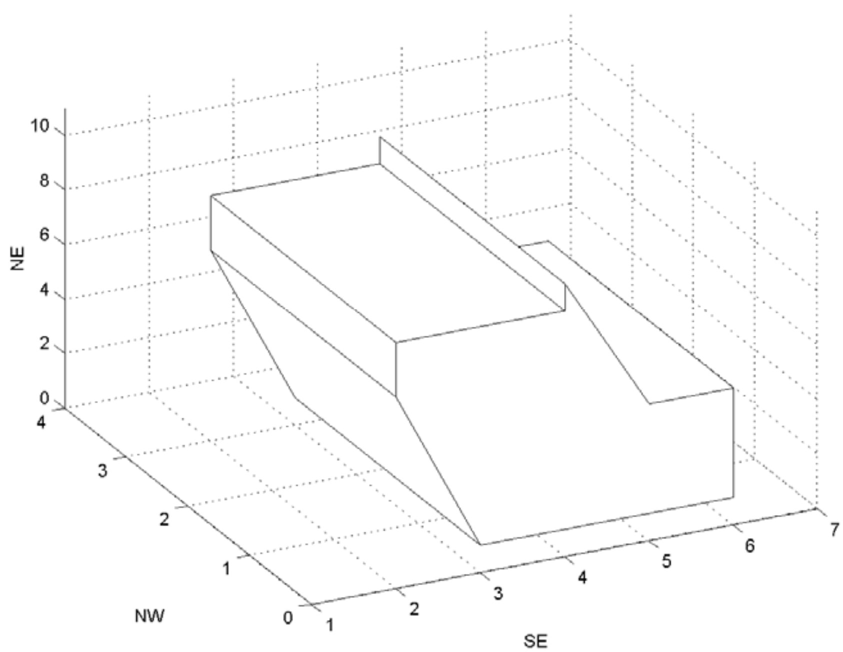

Figure 10. Reachable Set in the Vicinity of DFW Between 1212 and 1236 local time on a Typical Day