Blended Homing Guidance Law Using Fuzzy Logic

By

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Abstract

The application of fuzzy logic for the development of guidance laws for homing missiles is presented. Fuzzy logic approximation of the well known proportional navigation guidance law is discussed, followed by the development of a blended guidance law using fuzzy logic. The objective of the latter guidance law is to combine desirable features of three homing guidance laws to enhance the interception of targets performing uncertain maneuvers. Fuzzy logic guidance law development employs triangular, trapezoidal and sigmoidal membership functions. Mamdani-style inference is employed in the fuzzy inference system. Simulation results using point-mass missile model and a spiraling, high-speed ballistic target model are given.

Introduction

Naval vessels are subject to threats from enemy tactical ballistic missiles and sea-skimming missiles [1]. These threats may be fast and maneuverable, and may employ unknown evasive strategies and stealthy airframes. Engagements against these threats can occur over a wide range of flight conditions. The missile dynamic model may be highly uncertain at these flight conditions. Uncertainties can arise in aerodynamic data and can include previously unknown and unquantified coupling effects. Ship defense missile guidance and control systems must be capable of delivering agile performance while handling nonlinearities and uncertainties in the vehicle model, atmosphere, and ambient winds.

The focus of the present paper is on the development of fuzzy logic guidance laws capable of handling large uncertainties in the missile model. Two different guidance laws are discussed. The

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Presented at the 1998 AIAA Guidance, Navigation and Control Conference, August 10-12, Boston, MA

first guidance law approximates the well known proportional navigation guidance law [2] using fuzzy logic, while the second fuzzy logic scheme blends three guidance laws to obtain a composite guidance law. The composite guidance law development is motivated by the advantages of the mixed-strategy guidance strategies discussed in References 3 and 4. According to that research, mixed strategy guidance strategies are found to be more effective against targets performing uncertain maneuvers than “pure” guidance strategies such as proportional navigation.

Fuzzy logic [5] control has emerged in recent years as an effective methodology for the control of nonlinear, uncertain dynamic systems [6 - 18]. Fuzzy logic control concept employs a linguistic approach to control in that the instantaneous value of the control variables depend on the inference derived using a set of IF-Then-Else type rules. The rules are generally derived from state transition relationships. As a result, the fuzzy logic approach can treat linear, nonlinear, continuous and discrete-time systems using the same frame work. Moreover, since fuzzy logic is linguistically based, it can handle uncertainty descriptions that are much more general than those treated by robust control theory. Due to these factors, the fuzzy logic paradigm can help develop guidance systems that are based on a mix of qualitative and quantitative performance specifications.

The past several years have witnessed a rapid growth in the number and variety of applications of fuzzy logic. The most visible applications have been in the realm of consumer products, intelligent control, and industrial automation systems [6 - 11, 13, 14]. More recently, application of the fuzzy logic in a few flight control problems have been reported in the literature [15 -18]. Reference 15 has pointed-out that the fuzzy logic paradigm is most effective when used in a supervisory control role, such as guidance and blending of multiple actuators. In the missile actuator blending logic design problem, the use of fuzzy logic will permit the inclusion of qualitative performance requirements into the design process. Once the general qualitative features of the actuator blending logic are defined, the fuzzy logic system can be designed using a state space based fuzzy logic rule generation process [19].

The central elements of a fuzzy logic based missile control system can be examined with the help of Figure 1. Fuzzy logic control systems contain two distinct types of variables. The physical variables are the variables that are given as numerical values in terms of physical units. Actuator inputs and outputs, missile states and the sensor outputs are all physical variables. Linguistic variables are qualitative descriptions of the physical variables. For instance, a certain numerical value of fin deflection may be classified as being “high”, “medium”, “low” and “very high”. According to fuzzy logic theory [10], a physical variable can be correspond to several linguistic variables through the definition of “membership” functions. A membership function defines the degree to which a certain physical variable can be associated with a linguistic variable. Every fuzzy logic control system includes subsystems for converting physical variables into linguistic variables and vice versa. These are termed as the *Fuzzifier* and *Defuzzifier*, respectively. The linguistic variables form the inputs to a *Fuzzy Inference Engine* that uses a *Knowledge Base* to generate a set of linguistic outputs. The knowledge base consists of a collection of rules that associate each combination of the input linguistic variables into a set of desirable actions expressed in the form of output linguistic variables. The knowledge base incorporates all the known input-output behaviors of the dynamic system, uncertainties and qualitative design objectives. The fuzzy inference engine uses the knowledge base and the input linguistic variables to generate linguistic outputs. The defuzzifier then converts the linguistic variables into "crisp" actuator commands.

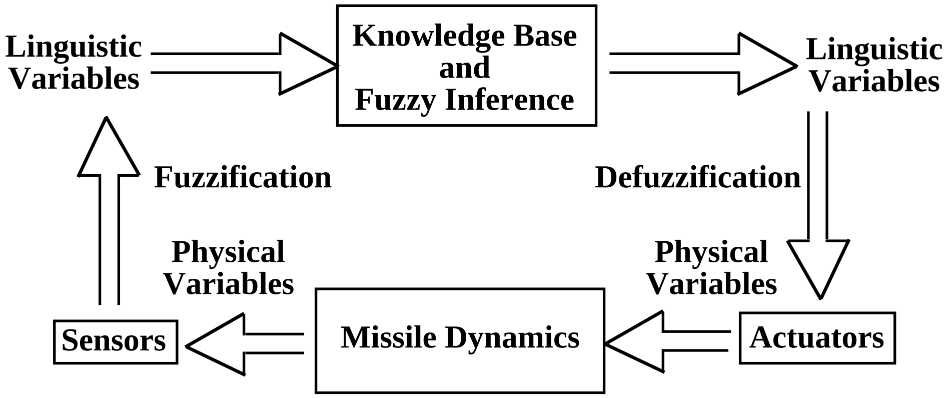


Fig. 1. The Structure of a Fuzzy Logic Control System

From the foregoing discussion, it can be seen that the task of designing fuzzy logic controllers is essentially one of selecting membership functions for the fuzzifier and defuzzifier, building a knowledge base, and setting up the fuzzy inference engine. The designer has a wide variety of choices in setting-up each of these functional subsystems. For instance, the knowledge base can be generated using state transition methodology described in Reference 19. After initial set up, the fuzzy logic rules can be further refined using adaptive algorithms as described in References 20 and 21. Several varieties of fuzzy membership functions for the fuzzy inference system inputs and outputs have been described in the literature; triangular, trapezoidal and sigmoidal functions being examples. The defuzzification methods can be based on center-of-area method [10], or the mean-of-the-maximum method [12].

The application of the fuzzy logic methodology for developing a fuzzy guidance system for an advanced missile is the focus of present paper. The present research employs the missile configuration data, models and autopilots discussed in Reference 22. The following sections will outline the missile model, the target model and fuzzy guidance laws.

2. Missile Model

A point-mass missile model is employed in the present research. The point-mass model incorporates the missile speed V, the flight path angle γ, the heading angle χ and the missile position vector components x, y, z as the state variables. The point-missile dynamics is described by the nonlinear differential equations:

$$\dot{V} = \frac{F_V}{m} - g\sin\gamma$$

$$\dot{\gamma} = \frac{F_\gamma}{mV} - g\cos\gamma$$

$$\dot{\chi} = \frac{F_\chi}{mV}$$

$$\dot{x} = V\cos\gamma\cos\chi$$

$$\dot{y} = V\cos\gamma\sin\chi$$

$$\dot{z} = -V\sin\gamma$$

F_V is the aerodynamic force along the velocity vector, F_γ and F_χ are the force components normal to the velocity vector. The variables commanded by the guidance law are the lateral acceleration components

$$a_z = \frac{F_\gamma}{m}, a_y = \frac{F_\chi}{m}.$$

The force component F_V is given by: $F_V = -C_A \bar{q} s$, with the axial force coefficient C_A being a function of the lateral acceleration components. The variable $\bar{q}$ is the dynamic pressure and s is the reference area.

3. Spiraling Target Model

References 23 and 24 have indicated that tactical ballistic missiles (TBM) can experience severe spiral maneuvers as they reenter earth's atmosphere. It has been estimated in References 3 and 4 that the spiral maneuvers could increase in magnitude from 1 to 10 g's as the vehicle descends in altitude from 100 kft to 60 kft. A spiraling frequency range of 0.5 to 1.0 Hz is expected. Based on these observations, a simple kinematic spiraling target model is set up for evaluating the guidance laws. The spiraling target model simulates a body falling under the action of gravity, with a rotating horizontal velocity component. The target motions are described in an earth-fixed inertial frame with the Z-axis pointing along the local gravity vector, X-axis pointing towards east, and the Y-axis completing the right-handed triad. The equations of motion of the target are given by:

$$\ddot{x}_T = -A\omega^2 \sin\omega t, \ddot{y}_T = -A\omega^2 \cos\omega t \text{ and } \ddot{z}_T = -g$$

Here, A is the amplitude of the spiral motion in feet, ω is the frequency of the spiral in radians/second, t is the elapsed time in seconds, and g is the acceleration due to gravity. Both the amplitude and frequency of the spiral are assumed to be constant in the present research.

Figures 2 and 3 illustrate sample trajectories of the target for two distinct spiraling frequencies. The initial conditions used for these trajectories are: x_{t0} = 5000 ft, x_{td0} = 100 ft/s, y_{t0} = 500 ft, y_{td0} = 100 ft/s, z_{t0} = -50500 ft, z_{td0} = 100 ft/s, A = 100 ft.

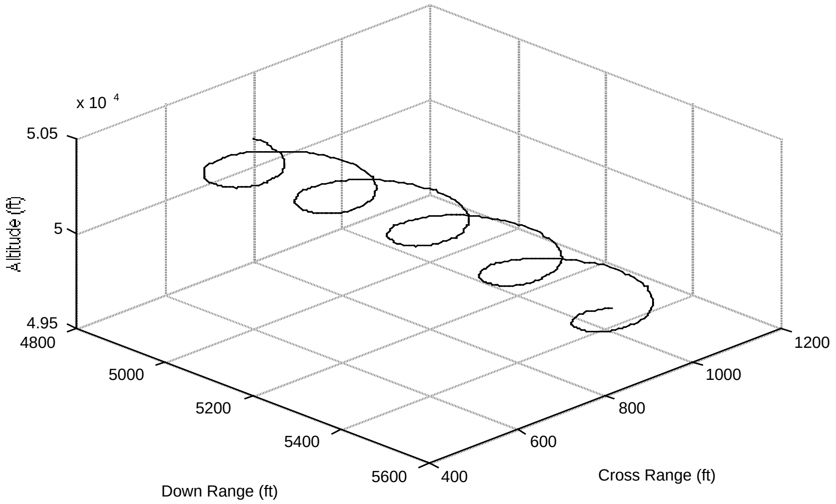


Fig. 2. Spiraling Target Trajectory

(Horizontal Velocity Component Rotational Frequency : 4 rad/s)

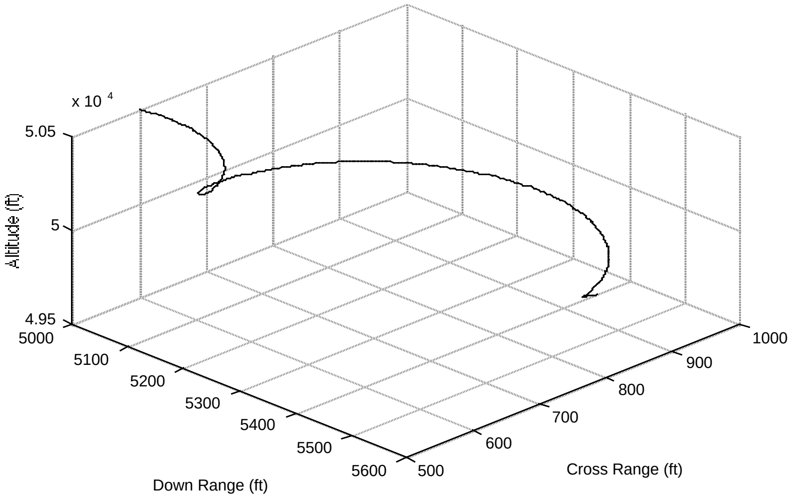


Fig. 3. Spiraling Target Trajectory

(Horizontal Velocity Component Rotational Frequency : 2 rad/s)

Fuzzy guidance laws described in the following sections will be evaluated using this spiraling target model.

4. Fuzzy Logic Guidance Laws

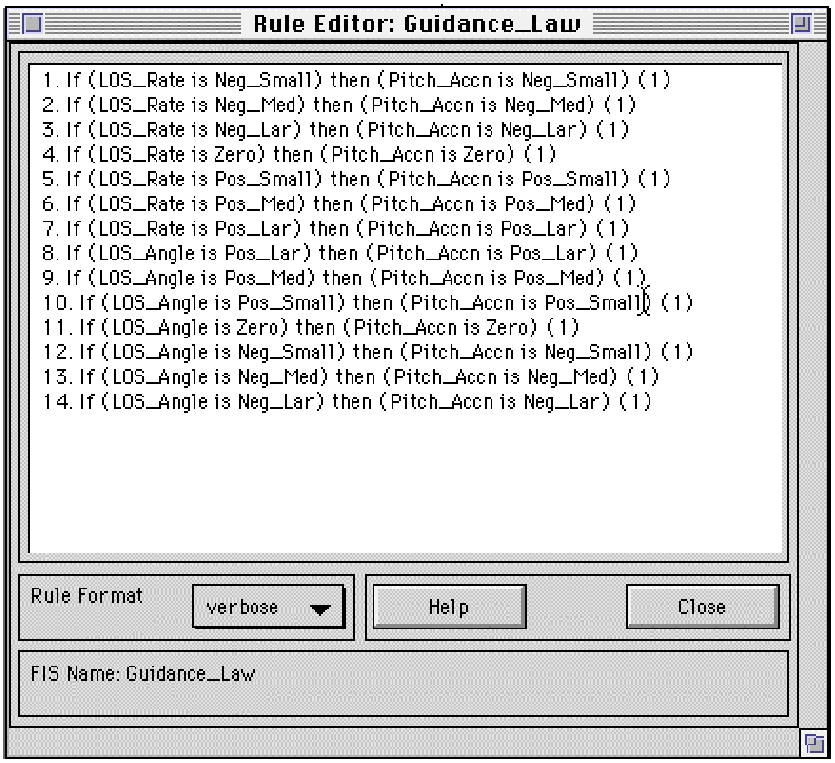
The guidance law generates steering commands to the missile in order to direct it towards the target to achieve interception. It uses the relative missile position/velocity information, and target acceleration information (if available) to generate the steering commands. The most common form of the steering commands are in the form of pitch and yaw acceleration components. The missile autopilot has the responsibility for tracking the commanded acceleration components. Thus, the guidance law can be considered to be a mapping between the target relative measurements and the steering commands.

In this section, fuzzy logic will be used to develop guidance laws. Two distinct fuzzy logic guidance laws will be discussed. The first guidance law uses the line-of-sight rate and line-of-sight angles to generate the steering commands. This fuzzy guidance law can be considered to be a member of the proportional navigation family. The second guidance law is a composite guidance scheme that uses fuzzy logic to blend three well known guidance laws to obtain enhanced homing performance. Fuzzy logic plays more of a supervisory role in the second guidance law. These guidance laws are then evaluated using a spiraling target model discussed in Section 3. Details of the guidance laws are discussed in the following sections.

4.1. Fuzzy Proportional Navigation Guidance Law

This fuzzy guidance law is based on the observation that the classical proportional navigation guidance law [2] achieves target interception using only the line-of-sight rate measurements. This fact implies that it should be possible to guide the missile towards the target by applying a few fuzzy logic rules on line-of-sight rate measurements. Note that proportional navigation guidance law and its derivatives have been used successfully in several missile programs. In addition to the line-of-sight rate, the guidance commands can also be made functions of the intstantaneous missile relative line-of-sight angle to provide some control over the aspect angle at interception. Achieving a specified aspect angle at interception may be important to maximize warhead effectiveness [25].

A fuzzy inference system is set up with four inputs and two outputs. The inputs to this inference system are the missile relative line-of-sight angles and rates along the pitch and yaw axes. The outputs are the pitch and yaw acceleration commands. Five triangular membership functions, and two trapezoidal membership functions are used at the inputs to convert the line-of-sight angles and rates into linguistic variables. The trapezoidal membership functions at the boundaries of the desired input range serve to provide “saturation” behavior whenever the input is high. Similarly, seven output membership functions are used in the fuzzy inference system. Fourteen fuzzy logic rules were set up, which are listed in Figure 4.



1. If (LOS_Rate is Neg_Small) then (Pitch_Accn is Neg_Small) (1)
2. If (LOS_Rate is Neg_Med) then (Pitch_Accn is Neg_Med) (1)
3. If (LOS_Rate is Neg_Lar) then (Pitch_Accn is Neg_Lar) (1)
4. If (LOS_Rate is Zero) then (Pitch_Accn is Zero) (1)
5. If (LOS_Rate is Pos_Small) then (Pitch_Accn is Pos_Small) (1)
6. If (LOS_Rate is Pos_Med) then (Pitch_Accn is Pos_Med) (1)
7. If (LOS_Rate is Pos_Lar) then (Pitch_Accn is Pos_Lar) (1)
8. If (LOS_Angle is Pos_Lar) then (Pitch_Accn is Pos_Lar) (1)
9. If (LOS_Angle is Pos_Med) then (Pitch_Accn is Pos_Med) (1)
10. If (LOS_Angle is Pos_Small) then (Pitch_Accn is Pos_Small) (1)
11. If (LOS_Angle is Zero) then (Pitch_Accn is Zero) (1)
12. If (LOS_Angle is Neg_Small) then (Pitch_Accn is Neg_Small) (1)
13. If (LOS_Angle is Neg_Med) then (Pitch_Accn is Neg_Med) (1)
14. If (LOS_Angle is Neg_Lar) then (Pitch_Accn is Neg_Lar) (1)

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Fig. 4. Fuzzy Logic Rules Used for Target Interception Guidance Law

The fuzzy guidance law is next evaluated in a point-mass missile simulation using the spiraling target model discussed in Section 3. The results of one of the simulation runs are given in Figures 5 through8. It may be observed that the guidance law is successful in bringing the missile within 10 feet of the target, at which point, the simulation was terminated.

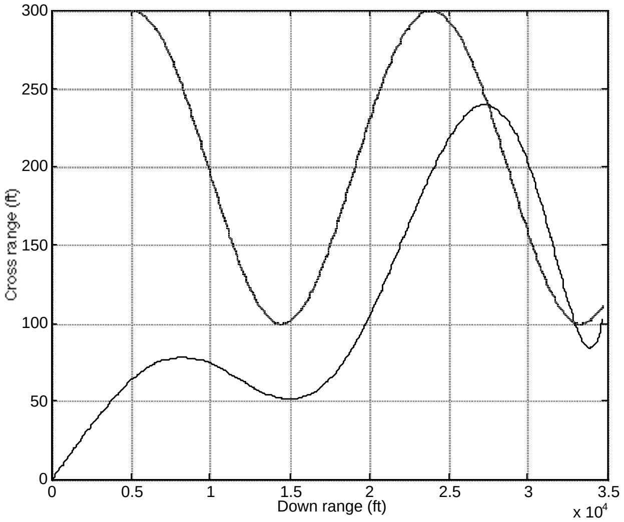


Fig. 5. Missile and Target Trajectories in the Horizontal Plane:

Fuzzy Proportional Navigation Guidance Law

(Solid Line: Missile, Dashed Line: Target)

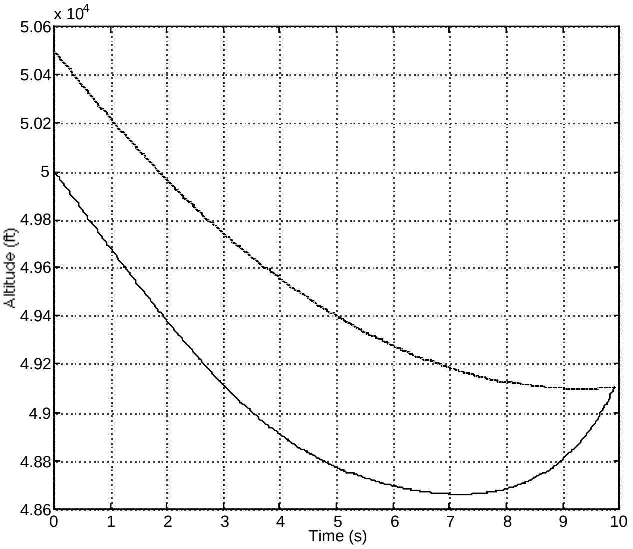


Fig. 6. Missile and Target Altitude History:

Fuzzy Proportional Navigation Guidance Law

(Solid Line: Missile, Dashed Line: Target)

The pitch/yaw acceleration commands given in Figure 7 and 8 show one of characteristic features shared by all proportional navigation guidance laws, namely, the large acceleration command as the missile gets closer to the target.

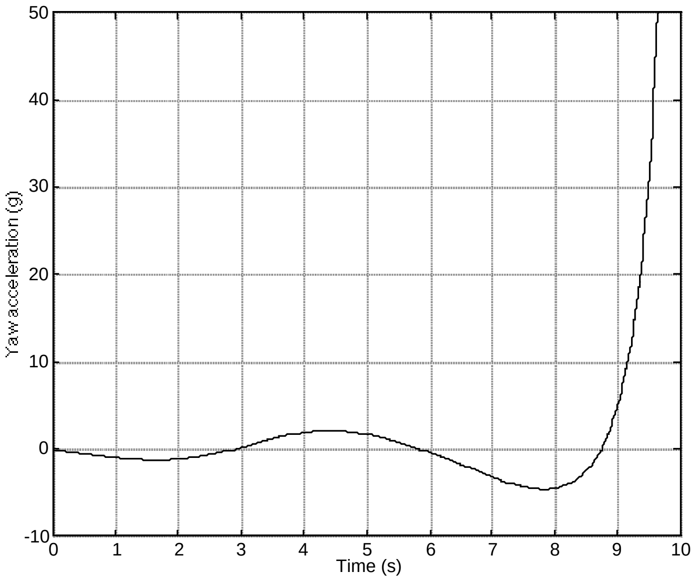


Fig. 7. Yaw Acceleration Command History

for the Fuzzy Proportional Navigation Guidance Law

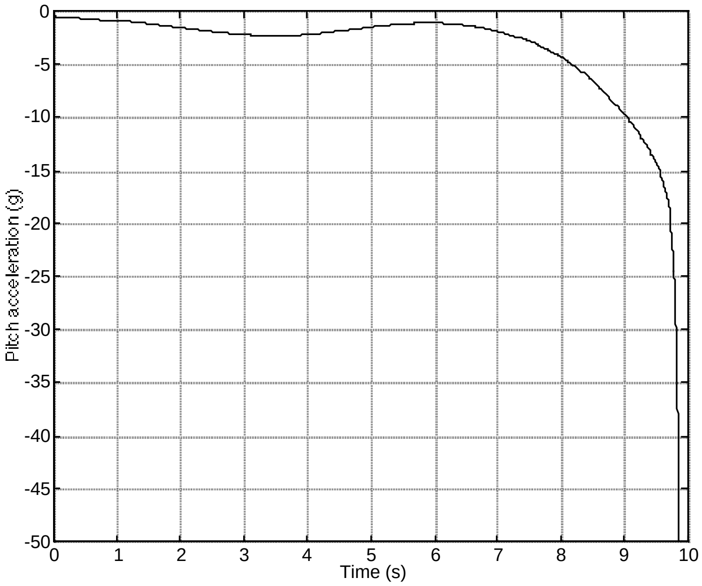


Fig. 8. Pitch Acceleration Command History

for the Fuzzy Proportional Navigation Guidance Law

Finally, the range-to-go history presented in Figure 9 shows a monotonic behavior. The miss distance at the termination of the simulation was about 10 feet.

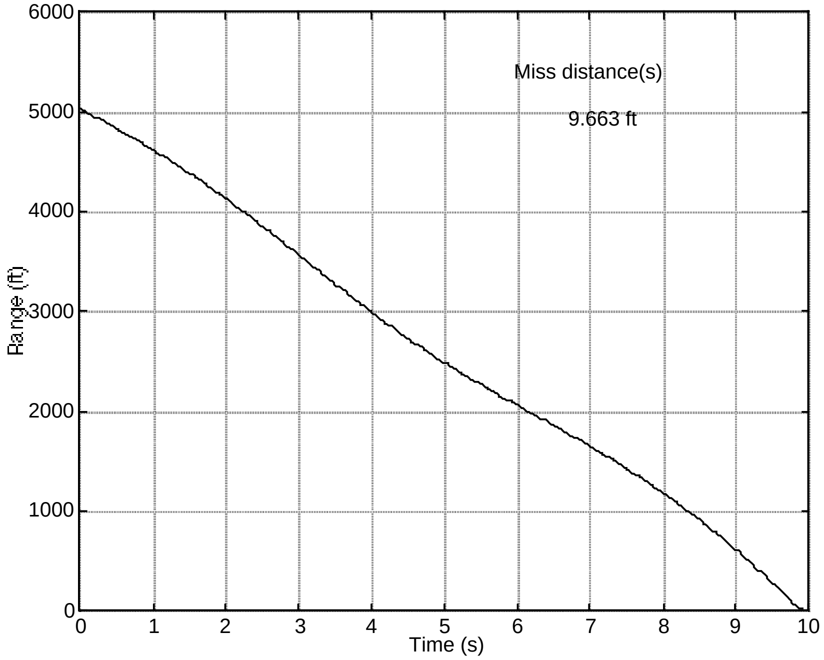


Fig. 9. Range to the Target using the

Fuzzy Proportional Navigation Guidance Law

It needs to be emphasized here that the fuzzy guidance lawpresented in this paper represents a very preliminary development effort. Its performance can be significantly enhanced by including additional fuzzy logic rules for modeling the target and missile dynamic behavior. An alternate approach to the development of fuzzy guidance laws will be presented in the following section.

5.3. A Composite Fuzzy Guidance Law

The composite fuzzy guidance law is based on the notion that each of the guidance laws reported in the literature [2] have a region of operation where they are superior to other guidance laws. Hence, if a fuzzy inference system could be set up that selects the appropriate guidance law based on the interception conditions, such a guidance scheme can be expected to incorporate the best features of all the guidance laws. Towards this end, three guidance laws are chosen for inclusion in the composite guidance law. These are the classical proportional navigation, bang-bang guidance law and the augmented proportional navigation.

The justification for selecting any one of the guidance laws from this set is based on the following heuristic analysis. When the target is far away, proportional navigation guidance law yields moderate acceleration commands. However, as the range becomes small, the acceleration commands will become very large. Thus, under small range conditions, alternate guidance laws that use bounded acceleration commands must be found. Since bang-bang guidance law intercepts target using bounded acceleration at every time instant, this guidance law is ideal for use under small range conditions. The augmented proportional navigation can serve as the "bridge" guidance scheme between proportional navigation and the bang-bang guidance law. Similar arguments can also be developed for guidance law selection under different range rate conditions.

The fuzzy inference scheme is next set up to implement this heuristic reasoning. Figure 10 shows the proposed composite fuzzy guidance scheme.

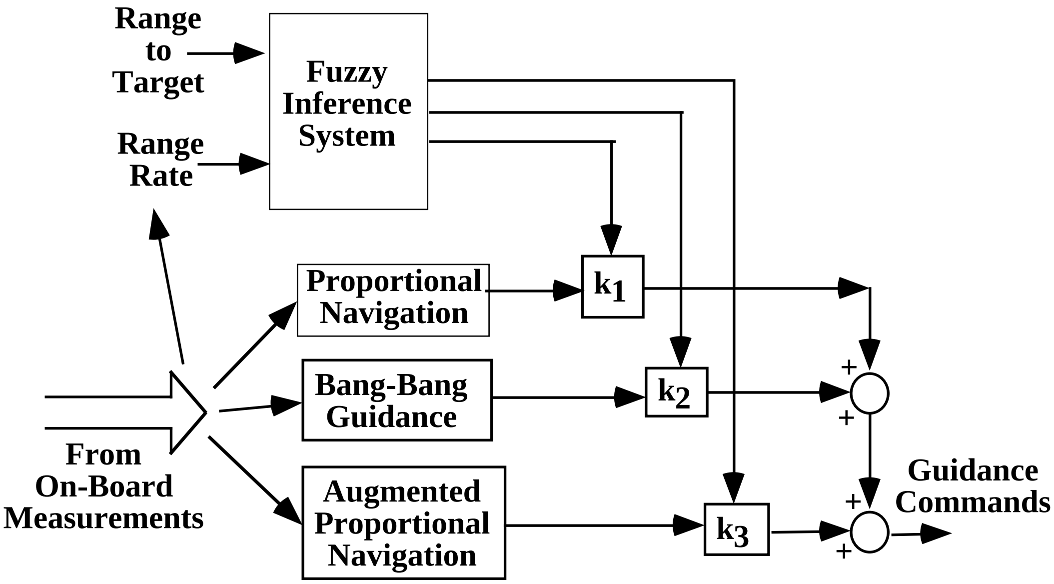
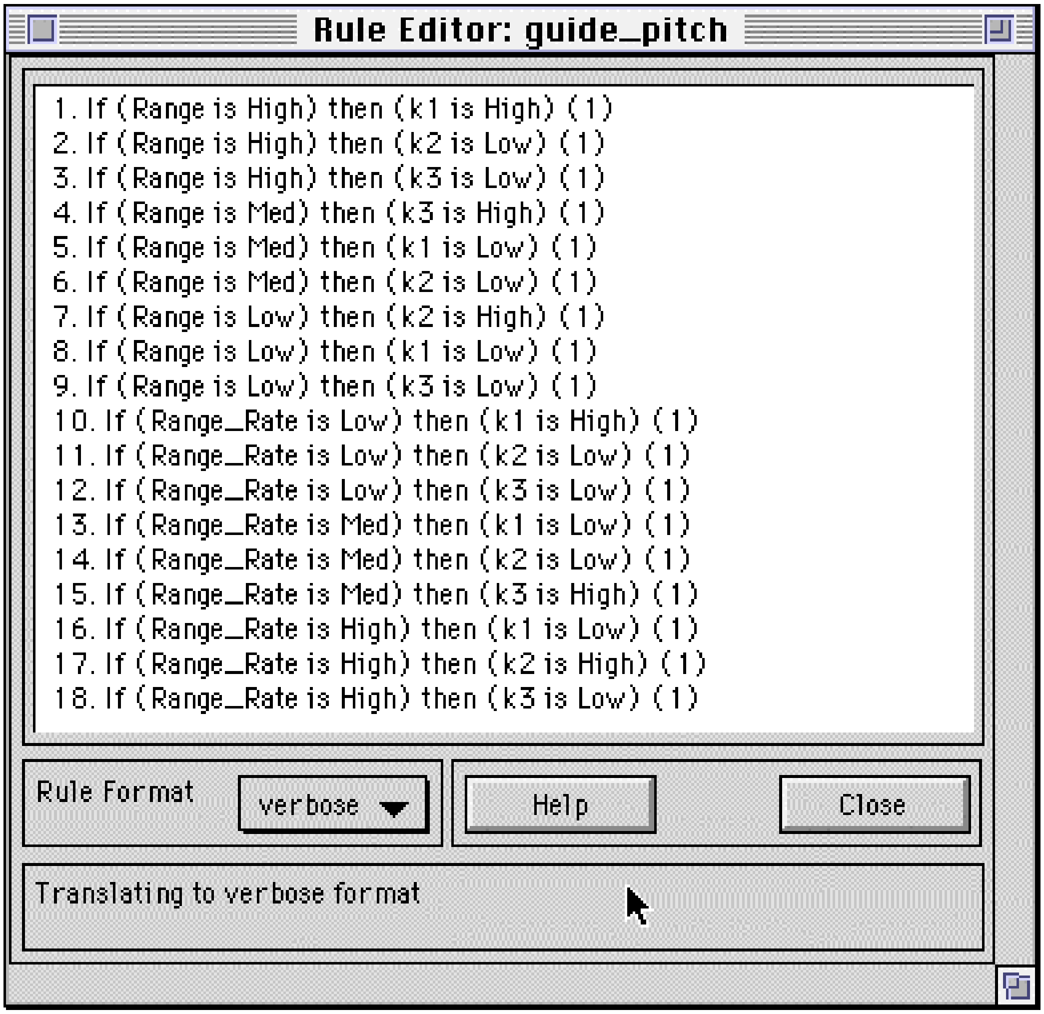


Fig. 10. Composite Fuzzy Guidance Law

In this figure, the fuzzy inference system selects the parameters k_1, k_2, k_3 to be between zero and one based on the measured range and range rate. Zero value indicates an inactive guidance logic, while a value between zero and one indicates the relative importance of the particular guidance law. Three triangular membership functions are used for each of the inputs of the fuzzy inference system. Each of the outputs of the fuzzy inference system uses two sigmoidal membership functions. Eighteen fuzzy inference rules are used in the composite fuzzy inference system. These rules are listed in Figure 11.

The composite guidance law is next evaluated in the point-mass missile simulation, together with the spiraling target model. The missile and target trajectories in the horizontal plane are given in Figure 12. This figure also shows the missile trajectories that would have resulted if the proportional navigation, bang-bang guidance law or augmented proportional navigation scheme were used individually. The corresponding altitude histories are given in Figure 13. The missile acceleration histories are given in Figure 14 and 15. It may be observed that the composite guidance law used proportional navigation during the first few seconds, and then switched to bang-bang control law. Towards the end, it used a combination of the guidance laws to achieve target intercept.

The heading angle and flight path angle histories for the missile while employing the composite fuzzy guidance scheme are given in Figure 16 and 17. Finally, the range-to-go is given as a function of time in Figure 18.



1. If (Range is High) then (k1 is High) (1)
2. If (Range is High) then (k2 is Low) (1)
3. If (Range is High) then (k3 is Low) (1)
4. If (Range is Med) then (k3 is High) (1)
5. If (Range is Med) then (k1 is Low) (1)
6. If (Range is Med) then (k2 is Low) (1)
7. If (Range is Low) then (k2 is High) (1)
8. If (Range is Low) then (k1 is Low) (1)
9. If (Range is Low) then (k3 is Low) (1)
10. If (Range_Rate is Low) then (k1 is High) (1)
11. If (Range_Rate is Low) then (k2 is Low) (1)
12. If (Range_Rate is Low) then (k3 is Low) (1)
13. If (Range_Rate is Med) then (k1 is Low) (1)
14. If (Range_Rate is Med) then (k2 is Low) (1)
15. If (Range_Rate is Med) then (k3 is High) (1)
16. If (Range_Rate is High) then (k1 is Low) (1)
17. If (Range_Rate is High) then (k2 is High) (1)
18. If (Range_Rate is High) then (k3 is Low) (1)

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Fig. 11. Fuzzy Logic Rules Used in the Composite Guidance Law

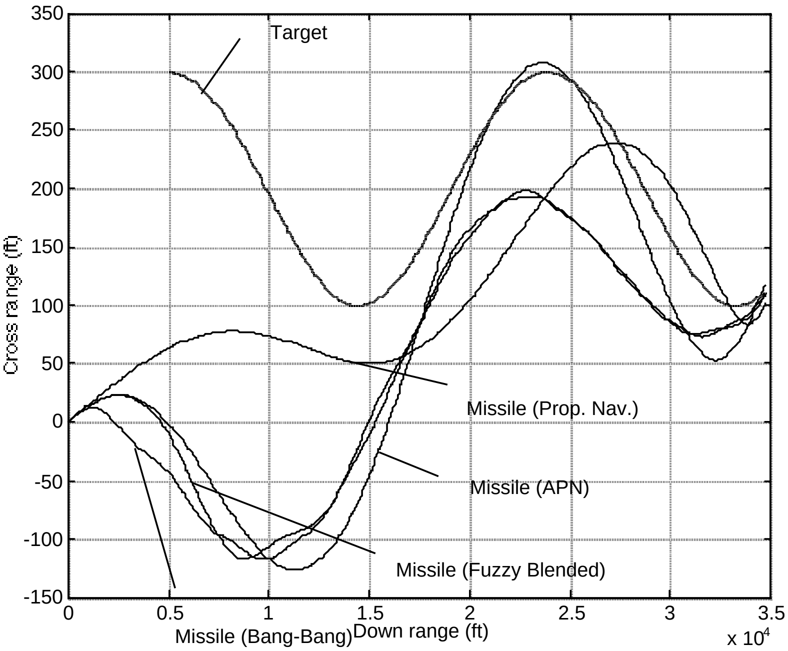

Fig. 12. Missile and Target Trajectories in the Horizontal Plane: Composite Fuzzy Guidance Scheme

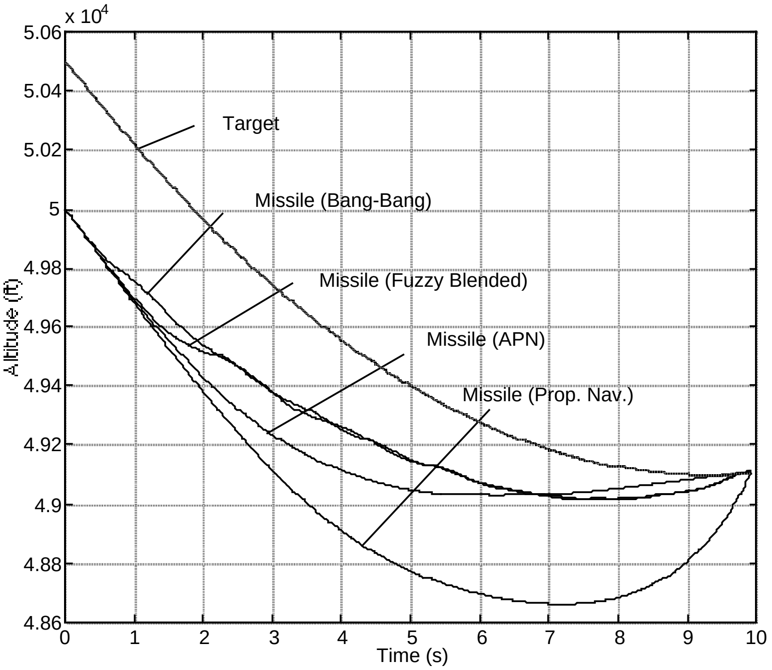

Fig. 13. Missile and Target Altitude Histories: Composite Fuzzy Guidance Scheme

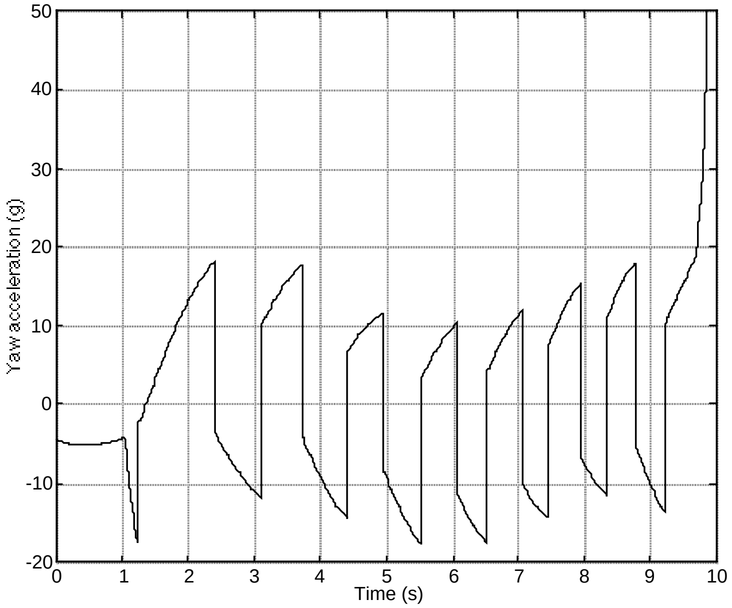

Fig. 14. Missile Yaw Acceleration History:

Composite Fuzzy Guidance Scheme

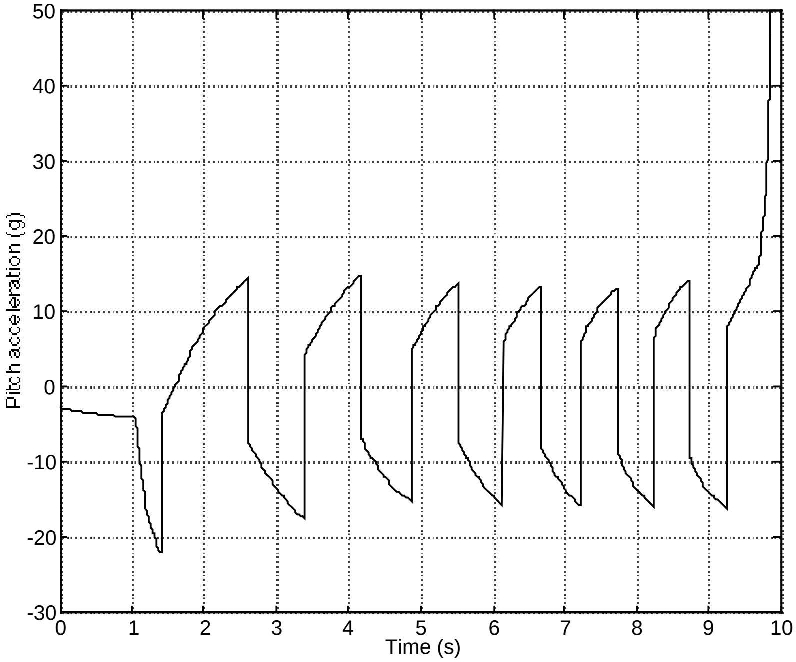

Fig. 15. Missile Pitch Acceleration History:

Composite Fuzzy Guidance Scheme

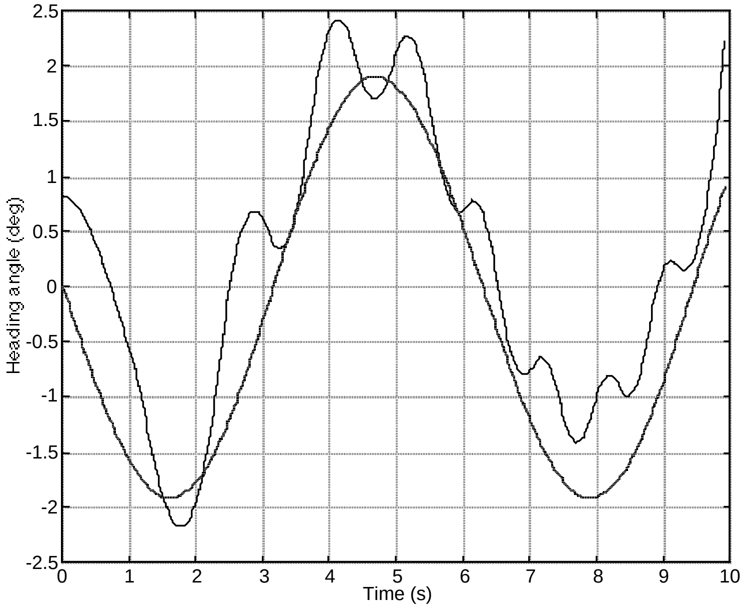

Fig. 16. Heading Angle Histories:

Composite Fuzzy Guidance Scheme

(Solid Line: Missile, Dashed Line: Target)

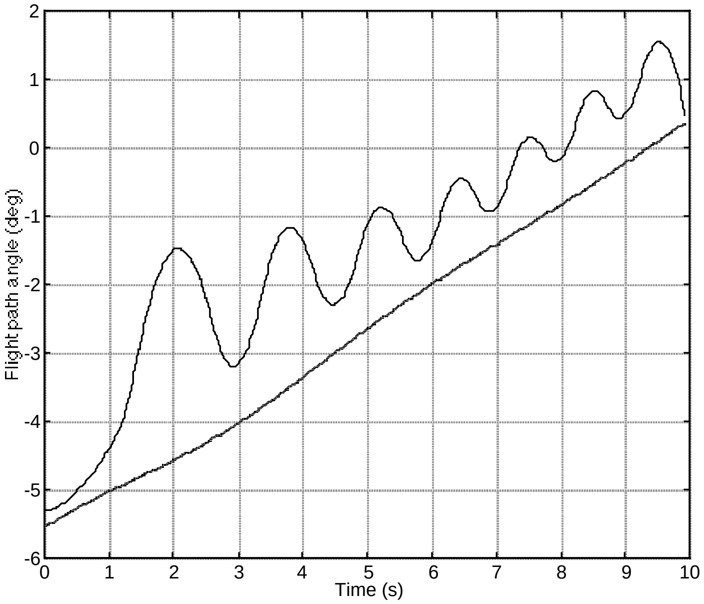

Fig. 17. Flight Path Angle Histories:

Composite Fuzzy Guidance Scheme

(Solid Line: Missile, Dashed Line: Target)

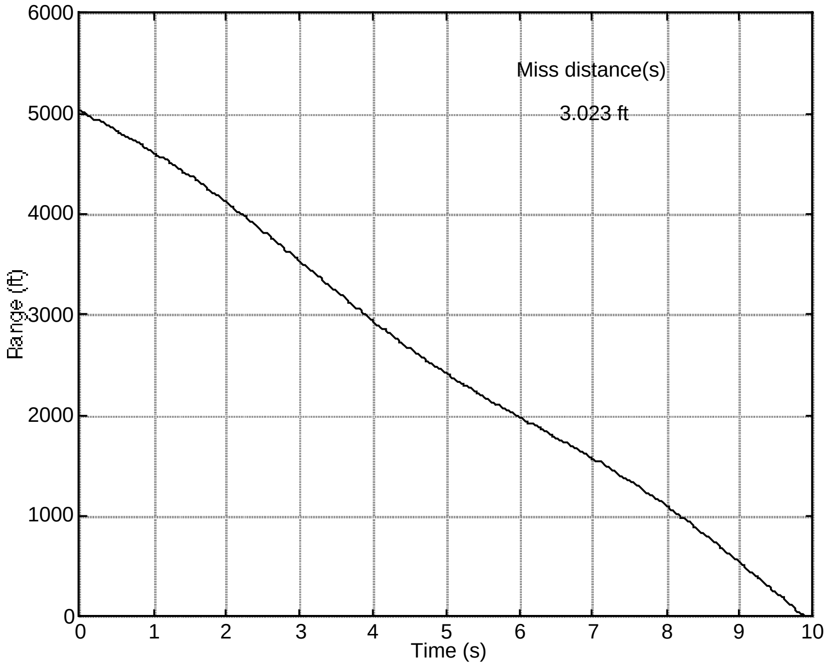

Fig. 18. Range to the Target using the Composite Fuzzy Guidance Scheme

Although the simulation results demonstrate the feasibility of synthesizing fuzzy guidance laws, the quantitative performance benefits of these guidance laws have not been established in this paper. During the present simulations, it became apparent that the fuzzy logic guidance laws can be further tuned to enhance performance. However, this process could not be completed during the present research effort due to a lack of time. During future research, the fuzzy logic guidance laws will be further generalized and compared with more traditional guidance schemes to assess the performance benefits. This process will require several simulation runs under different target maneuver and noise conditions.

Conclusions

This paper presented the development of two fuzzy logic guidance laws. The first guidance law is a fuzzy logic approximation of the proportional navigation guidance scheme. The second guidance law employed fuzzy logic for blending three diffrent homing guidance laws. The performance of both guidance laws were illustrated using a rigid-body missile model and a point-mass spiraling target model.

Fuzzy logic approach to the homing missile guidance problem allows the analyst to combine qualitative and quantitative aspects of the guidance task. Moreover, it permits the consideration of the missile model and target maneuver uncertainties in a qualitative manner.

Acknowledgement

This research was supported under Navy Contract No. N00178-97-C-3005 with Mr. John E. Bibel of Naval Surface Warfare Center, Dahlgren, VA serving as the technical monitor. The authors would like to thank Mr. Bibel, Mr. Ernest J. Ohlmeyer and Mr. D. Stephen Malyevac for several discussions on the missile models and the guidance problem.

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